Strolling with Euler: Trying to be Flat

We are going to call a graph “planar” if it is a diagram in which no edges cross. Draw a few planar graphs below.

Next, draw a few graphs that aren’t planar.

Often, we can turn graphs that look like they aren’t planar into planar graphs, by making an edge curved, or by moving a vertex (I will give examples on the board). Can you turn any of the graphs that you wrote above into a planar graph now?

On the board, I'll draw a picture of "K_5" and "$K_{3,3}$". In the space below, try to draw them without crossing any edges. Try at least four times each.

Now, for each of the planar graphs you drew on the first page, try calculating the following number:

(# of vertices) – (# of edges) + (# of faces).

In this case, a face is a region surrounded by non-crossing edges. What do you notice?

And now, explain why there is at least twice as many edges in any planar graph as there are faces:

Now, we will check and see what happens for "K_5." Take notes on the board presentation so that you have them for later.

For "$K_{3,3}$", we can see that there are no triangles in it. Then, if we draw it without crossing edges, every face would have at least four edges. Explain below why any planar graph with no triangles should have $4f \leq 2e$, and use this to show why "$K_{3,3}$" can't be planar.

Homework: Find out how to arrange the vertices and edges of "K_5" and "$K_{3,3}$" in 3D space so that there are no crossings.