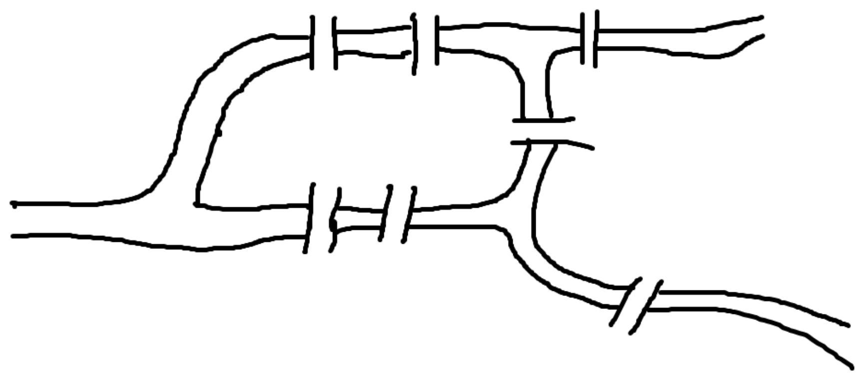
Strolling with Euler: Königsberg

Is it possible to find a path through the city below that crosses every bridge exactly once?



The above picture can be redrawn as a series of islands (vertices) and bridges connecting those islands (edges). Each bridge, of course, is between two islands.

Below, draw a simpler picture of the bridge scenario above:

Useful vocabulary: we call the number of edges coming from each vertex V the **degree of V**, deg(V). Now—is it possible to make the desired path over all the bridges?

Now suppose that we have the following variation. The Red Prince lives on the south bank; the Blue Prince lives on the north bank; on the east bank is a Church; in the small middle island is an Inn.

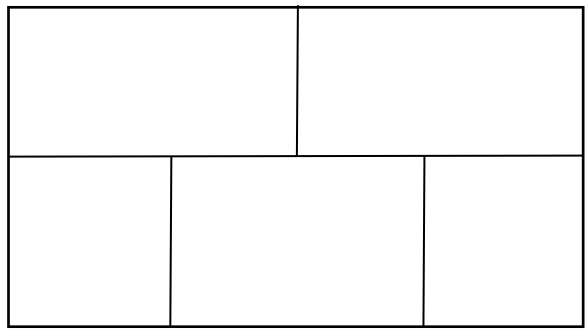
1. The Blue Prince would like to construct a bridge so that he can start at home, walk all the bridges, and finish at the inn—in such a way that the Red Prince cannot enjoy the same feat. Where should he build the eighth bridge?

2. The Red Prince would now like to build a ninth bridge so that *he* can start at home, walk the bridges, and finish at the inn—in such a way that ruins the Blue Prince's path. Where should he build the ninth bridge?

3. The Bishop at the church has been frustrated by all of the competitiveness and bridge-building. He would like to construct a 10^{th} bridge so that all of the town's inhabitants can leave home, walk as they like, and then end up back in bed. Where should the tenth bridge be placed?

We can also use this kind of thinking to solve the "5-room puzzle."

We would like to cross every line segment in the below diagram once without lifting our pens (the external segments that make up the big rectangle are ok, too). Is it possible?



For students who could use a little bit more to do:

Prove that the sum of the degrees of all the vertices is twice the number of edges in any graph.

At a party, guests greet each other by shaking hands. Prove that the number of guests who shake hands an *odd* number of times must be *even*.