Counting: Keeping things in order

You may remember the answer to the question:

How many ways are there to arrange three different pieces of fruit: an apple, an orange, a banana?

Draw the different arrangements below:

There are _____ arrangements.

Your older sister tells you that a tomato is also a kind of fruit, and that you should arrange it *also*.

How many ways are there to arrange the apple, orange, and banana from the first part, along with the tomato?

Draw the different arrangements below:

There are _____ arrangements.

There is a relationship between your answers to the first and second parts. Using that relationship, we can find a rule that would allow us to arrange any number **n** of objects. Write it below:

The special notation for rearranging **n** different objects is: _____

Try these problems:

1. A pets magazine wants to publish articles on 13 different pets this season. If the company has already decided which pets will be featured, how many different ways are there to order the pets, one per week?

    a. What if they have already decided to feature kittens first and hedgehogs last?

    b. What if they want instead to feature the canaries before the ducks?

2. How many 9-digit numbers can be obtained by using each of the digits 1, 2, … 9 exactly once? How many of these are bigger than 600,000,000?

3. A phone number in the United States consists of three digits (the first one being non-zero)—the area code—followed by seven more digits, with the first being non-zero.
    a. How many possible phone numbers are there?

    b. How many possible phone numbers are there with area code (510)?

    c. With area code (510) or (415)?

One last situation. Suppose that you would now like to rearrange two oranges, two apples, and a banana. You can't tell the difference between the oranges, nor between the apples.

Try drawing the situation below.

The number of rearrangements is:

In this case, the oranges are **indistinct**, as are the apples. We will divide the total number of rearrangements by the number of rearrangements of the oranges (2) and the number of rearrangements of the apples (2), because the situations in which the apples are exchanged with each other don't matter.

Try this problem: How many ways are there to arrange on the shelf 3 copies of an algebra book, 4 copies of a geometry book, and all 4 different volumes of Winnie-The-Pooh?

Homework:

The number of ways to rearrange the letters in COUNTING is:

The number of ways to rearrange the letters in REARRANGE is:

The number of ways to rearrange the letters in COMBINATORICS is: