PLACE VALUE CHALLENGES

Here's a selection of problems involving place value. These are all base ten problems (no exotic bases this week). Most of the problems are from Dr. Harold Reiter's math handouts.

1. Consider the number $N = 123456789101112...5960$, obtained by writing the numbers from 1 to 60 in order next to one another. What is the largest number that can be produced by crossing out 100 digits of N? What is the smallest number that can be produced by crossing out 100 digits of N?

2. Pick a three digit number. Multiply it by 7. Then multiply your answer by 11, and finally multiply by 13. Explain why you got that answer.

3. Take the first three digits of your phone number (NOT the Area code...) Then multiply by 80. Next add 1. Multiply by 250. Then add to this the last 4 digits of your phone number. Then add to this the last 4 digits of your phone number again. Subtract 250. Finally divide number by 2. What do you get and why does this work? Explain why you get such an interesting answer.

4. A two-digit number is 6 more than 4 times the sum of its digits. The digits from left to right are consecutive even integers. Find the number.

5. Use each of the five digits 1, 3, 5, 7 and 9 exactly once to build two numbers A and B such that $A \cdot B$ is as large as possible. Then build two numbers C and D such that $C \times D$ is as small as possible.

6. Next, consider the following problem. Find a four-digit number $\underline{abcd}$ which is reversed when multiplied by 9. In other words, find digits a, b, c, and d such that

$$9 \cdot \underline{abcd} = \underline{dcba}.$$

   (The underline notation is how we indicate that we have a string of digits, rather than the **product** $abcd = a \cdot b \cdot c \cdot d$.)

7. Suppose a, b, c, and d are digits and the sum of the two four-digit numbers $\underline{abcd}$ and $\underline{dabc}$ is 6017. Find all four-digit numbers $\underline{abcd}$ with this property. Note that $\underline{abcd}$ is a four-digit number only if $a \neq 0$.

8. What is the largest 5-digit multiple of 11 that has exactly 3 different digits? (Hint: the divisibility trick for 11's is to test whether the alternating sum of digits is divisible by 11. For example, consider 4737688516. The alternating sum of digits is $4-7+3-7+6-8+8-5+1-6 = -11$, which IS divisible by 11, so the larger number is too.)