GAUSSIAN INTEGER KENKEN

Nothing says KenKen variants must use plain old integers! The symbol $\mathbb{Z}[i]$ denotes the *Gaussian integers*, i.e. combinations of integers and integer multiples of the complex number $i = \sqrt{-1}$. That is $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}\}$.

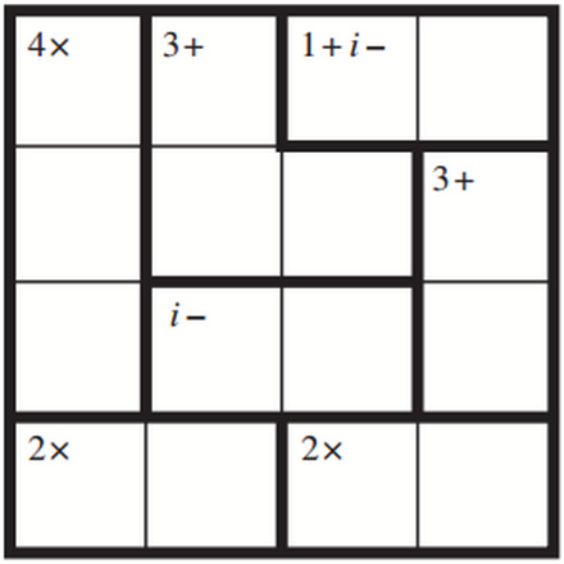
In the Gaussian integers, some things we took for granted before are now a bit crazy. Even though 2 is a prime number in $\mathbb{Z}$, there are multiple ways to factor it in $\mathbb{Z}[i]$.
For example, $2 = 1 \cdot 2$, but also $2 = (1 + i)(1 - i)$.

Before you can make good progress on Gaussian integer KenKen, you should probably brush up on Gaussian integer multiplication. The set $\{1, 2, 1 + i, 1 - 2, 1 + 2i, 1 - 2i\}$ is just a small subset of $\mathbb{Z}[i]$, but it's enough to make some nice 6×6 KenKen puzzles. Before you get started, make an addition table, and a multiplication table for this set. (Some of the results from addition and multiplication may land outside this set, but that's not a problem here.)

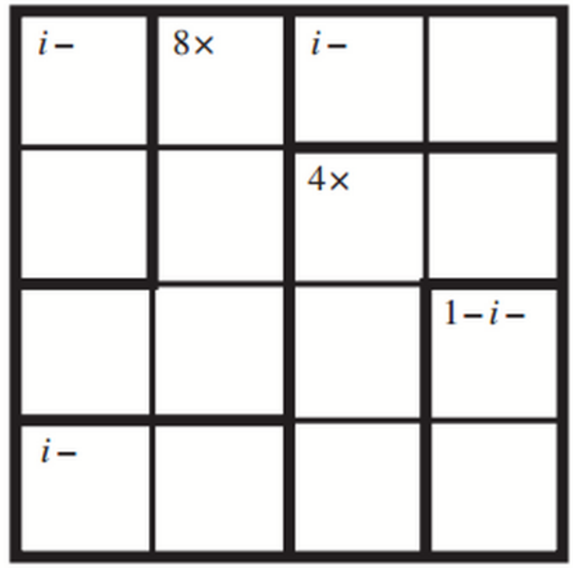
+	1	2	1+i	1-i	1+2i	1-2i
1						
2						
1+i						
1-i						
1+2i						
1-2i						

×	1	2	1+i	1-i	1+2i	1-2i
1						
2						
1+i						
1-i						
1+2i						
1-2i						

The puzzles on the next page are from John J. Watkins' paper *Triangular Numbers, Gaussian Integers, and KenKen*. Once you get a bit of practice solving them, can you create your own 4×4 or 6×6 Gaussian integer KenKen puzzle?

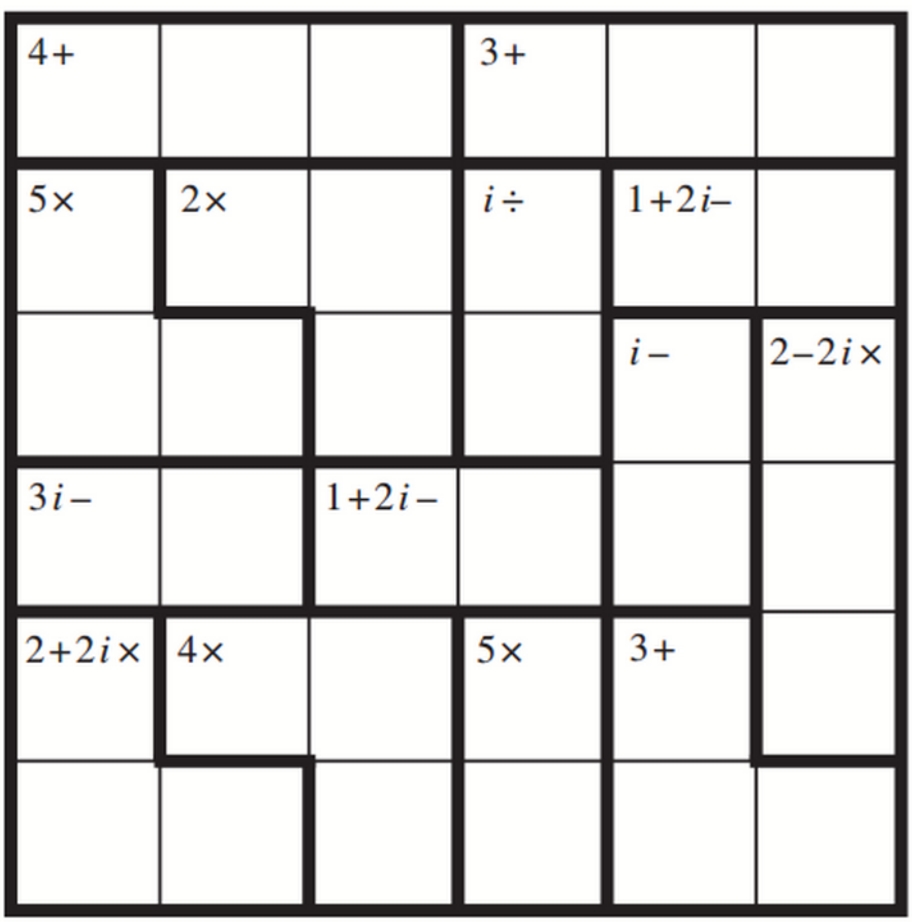


(d)



(e)

Figure 4. Solve using the four numbers 1, $1+i$, $1-i$, and 2.



(f)

Figure 5. Solve using the six numbers 1, $1+i$, $1-i$, $1+2i$, $1-2i$, and 2.