

## Valentine's Day Contest Problems

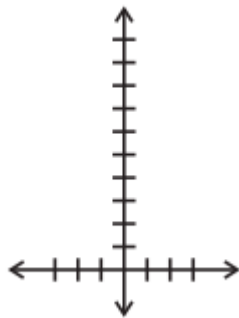
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Today's session is dedicated to exploring a variety of contest problems you might find. Students can work together in groups on the Valentine's Day problem set provided. We will work in small groups and then present problems towards the end of the session.

## Exercises

1. Sherry is making valentines for her classmates. Each valentine is one color (red, pink or white) and has one message written on it ("Be Mine," "Awesome," "Cutie" or "Luv Ya"). How many distinct valentines can she make?
2. Of the four statements given below about a natural number  $\heartsuit$ , two are true and two are false.
  - $\heartsuit$  is divisible by 5
  - $\heartsuit$  is divisible by 11
  - $\heartsuit$  is divisible by 55
  - $\heartsuit$  is less than 10What is the number  $\heartsuit$ ?
3. According to "A History of Valentine's Day Cards in America" by T.M. Wilson, in 1847 Esther Howland was the first person to mass-produce Valentine's Day cards. She made them out of lace, paint and expensive paper, and each one was individually written by a skilled calligrapher. The average card sold for \$7.50 while others cost as much as \$50. If 10 cents in 1847 would be equivalent to \$1.85 today, how much would the average card and most expensive card have cost today?
4. Kelly decided to celebrate Valentine's Day for an entire month. She started giving her Valentine 1 candy heart on Jan. 14, 2 candy hearts on Jan. 15, 4 candy hearts on Jan. 16, and continued doubling the amount of hearts each day through Feb. 14. If 200 candy hearts come in a bag, how many bags of candy hearts would Kelly need just for Feb. 14?
5. For Valentine's Day Ethan wanted to send Mary Beth 11 balloons since that was her favorite number. In the store, plain-colored balloons cost \$0.75 each, multi-colored balloons cost \$1.30 each, and extra-large balloons cost \$1.50 each. How many different combinations of 11 balloons can Ethan buy if he has only \$12.00?
6. How many divisors does 222,016 have?
7. Kia decided that she wants to give each of her friends a small pouch of candy hearts. She'll use fabric and ribbon to make and tie the pouch, and each one will contain four candy hearts. The hearts come in six colors: white, orange, pink, green, purple and yellow. If each pouch contains exactly four hearts, such that no two hearts are the same color, how many possible combinations of hearts could be in a pouch?

8. Mrs. Stuver’s art class has used geometric shapes to design a valentine in the shape of a heart. They have placed two adjacent semicircles along one side of an equilateral triangle so that the diameters of the semicircles and one side of the triangle are concurrent. The diameter of each semicircle is exactly one-half the length of the side of the triangle. The length of each side of the triangle is 4 inches. What is the area of the valentine in square inches? Express your answer as a decimal to the nearest tenth.
9. Mrs. Stuver’s class then decorated the perimeter of each heart-shaped valentine with lace. What is the length, in inches, of the perimeter of the valentine? Express your answer as a decimal to the nearest tenth.
10. Two pink valentines and two green valentines are delivered at random to two girls and two boys so that each girl and each boy receives exactly one valentine. What is the probability that each girl receives a pink valentine and each boy receives a green valentine? Express your answer as a common fraction.
11. On some graph paper, graph the following segments:



- $y = x$  for  $0 \leq x \leq 2$
- $y = 2x - 2$  for  $2 \leq x \leq 3$
- $x = 3$  for  $4 \leq y \leq 6$
- $y = -x + 9$  for  $2 \leq x \leq 3$
- $y = 7$  for  $1 \leq x \leq 2$
- $y = x + 6$  for  $0 \leq x \leq 1$

Now reflect each segment over the  $y$ -axis. What popular shape have you drawn?

12. Let  $S = \sqrt{14 + \sqrt{14 + \sqrt{14 + \dots}}}$ . Find the exact value of  $S$ .

## 1 Challenge of the Day

Denote  $m \heartsuit n = \frac{mn+4}{m+n}$ . Find the value of

$$((((2016 \heartsuit 2015) \heartsuit 2014) \heartsuit \dots \heartsuit 2) \heartsuit 1) \heartsuit 0.$$