AMC 8/10: Principles and Practice

November 3$^{\text{rd}}$ 2015

Set 1: Numbers of Numbers

(**A**) The average of the five numbers in a list is 54. The average of the first two numbers is 48. What is the average of the last three numbers?

(**B**) Find the number of two-digit positive integers whose digits total 7.

(**C**) Let R be a set of nine distinct integers. Six of the elements are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of R?

(**D**) For how many positive integer values of n are both $\frac{n}{3}$ and $3n$ three-digit whole numbers?

(**E**) What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

(**F**) Penniless Pete's piggy bank has no pennies in it, but it has 100 coins, all nickels, dimes, and quarters, whose total value is \$8.35. It does not necessarily contain coins of all three types. What is the difference between the largest and smallest number of dimes that could be in the bank?

Set 2: Calculations

(**A**) Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they shared, how many meals should they have ordered to have enough food for the 12 of them?

(**B**) What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, $\frac{9}{23}$?

(**C**) After Sally takes 20 shots, she has made 55% of her shots. After she takes 5 more shots, she raises her percentage to 56%. How many of the last 5 shots did she make?

(**D**) Ms. Osborne asks each student in her class to draw a rectangle with integer side lengths and a perimeter of 50 units. What is the difference between the largest and smallest possible areas of the rectangles?

(**E**) What is the product of all the roots of the equation $\sqrt{5|x| + 8} = \sqrt{x^2 - 16}$?

(**F**) It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to her home along the same route. What is her average speed, in km/hr, for the round trip?

Set 3: Hidden Algebra

(**A**) Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

(**B**) What is the units digit of 13^{2012}?

(**C**) The sum of three numbers is 20. The first is four times the sum of the other two. The second is seven times the third. What is the product of all three?

(**D**) Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs $1 more than a pink pill, and Al's pills cost a total of $546 for the two weeks. How much does one green pill cost?

(**E**) If $log(xy^3) = 1$ and $log(x^2y) = 1$, what is $log(xy)$?

(**F**) In base 10, the number 2013 ends in the digit 3. In base 9, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

Set 4: Probable Problems

(A) Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

(B) At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is $\frac{2}{5}$. What fraction of the people in the room are married men?

(C) Three points are chosen randomly and independently on a circle. What is the probability that all three pairwise distances between the points are less than the radius of the circle?

(D) Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

(E) Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

(F) In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two point shots. Shenille attempted 30 shots. How many points did she score?