BMC Worksheet: Turning Lies into Truth

1. Suppose that a liar claims that his friend Joseph is telling the truth. What does this statement mean?
2. If a liar declares that a certain marble is black, what does this mean?
3. If a liar claims that there are no girls in the room, what would the true statement be?
4. Suppose a liar tells you, "My grandfather is at least 100 years old." What would be the true age of the grandfather?
5. Suppose the Big Bad Wolf declares, "There are at least 10 pigs in this house." If you know that the wolf is a liar, what would be the true statement about the pigs in the house?
6. The Big Bad Wolf complains to you, "The pig I ate last night weighed less than 10 pounds." If you know that the wolf is lying about *the pig's weight*, what would be a true statement? (All pigs weigh an integer number of pounds.)
7. Suppose a student claims that all students in his class are boys. If you know that this student is lying, what can you say about the number of boys in the class?

There is an island far away, whose inhabitants are quite unusual. Some of them never tell a lie, and others never utter a true statement. Those that who never lie call themselves *Knights*. Those who never tell the truth are known as *Liars*. Even though they have different personalities, knights and liars cannot be told apart by appearance: they wear the same types of clothes, and have the same haircuts.

Sometimes tourists visit this island. The *Tourists* are regular people: sometimes they lie, and sometimes they tell the truth. A few of the following problems (indicated by "*") are about the inhabitants of this island.

8. * While visiting the Knights and Liars Island, you meet a boy who tells you that he is a liar. Does he live on the island or is he a tourist?
9. * Two island boys, Sam and Bob, are introducing themselves to you. Sam says, "At least one of us is a liar." Can you figure out who is what?
10. * While visiting the Knights and Liars Island, I had a conversation with a local knight. I asked him the same question twice, and he gave me two different answers. What was my question?
11. * While visiting the island, you meet a group of three islanders: Tom, George and Betty. You ask each of them the same question: "How many knights are in your group?" Tom replies, "None." George says, "One." Can you figure out what Betty is? What is her response?
12. * While visiting the Knights and Liars Island, you meet two islanders, James and Peter. James tells you that at least one of the two is a liar. Are James and Peter knights or liars?
13. Thirty children came to a party. Out of any 12 of them, at least one is a boy. Out of any 20 of them, at least one is a girl. How many boys and how many girls are there at the party?

14. * While visiting the Knights and Liars Island, you come to a party. Every single person at this party tells you that there are some Liars in the room. What is really happening? How many Knights and Liars are at this party? (Remember that there can also be tourists, such as you. Tourists sometimes lie and sometimes tell the truth.)

15. * While visiting the Knights and Liars Island, you pass a beautiful garden where three islanders, Sam, Bob and Tom, are watching the sunset. You ask Sam, "Are you a Knight or a Liar?" Sam is shy; you cannot hear his quiet answer. So you ask Bob, "What did Sam say?" Bob answers, that "He said that he is a Liar." "Don't trust Bob! Bob is a liar!" screams Tom. Can you decide whether Bob and Tom are Knights or Liars?

16. * On the Island of Knights and Liars, Knights and Liars live in two separate villages. However, inhabitants of one village often visit their friends in the other village.

    This week, the island newspaper announced a contest: to come up with a single yes/no question that would allow a tourist to figure out whether he is in a knights' village or a liars' village. The question should be such that it can be addressed to anyone the tourist sees. It is not known in advance whether this local is a knight or a liar and whether he is a resident of this village or not. (There are no tourists other than the one asking the question.)

    Gregory, the Math Circle student, won the prize. What was his question?

    How would you modify the question if there were multiple villages of each type?

17. Mother says to Max, "All fish love to swim." Max replies, "I love to swim; therefore, I am a fish." Is Max correct? Why or why not?

18. Bim always tells the truth, and Bom always lies. What question could you ask each one of them if you want to get the same answer? (Assume that you don't know who does what.)

19. * Emma and Rachel are from the Island of Knights and Liars. One of them is a Liar and the other is a Knight. Emma claims that 2 rubies are more expensive than 3 sapphires. Rachel says that 3 rubies are more expensive than 4 sapphires. Is it the case that 12 rubies are more expensive than 18 sapphires?

20. * Abby, Ben, Chris and Dan are all inhabitants of the Island of Knights and Liars. Abby claims that Ben is a lier. Dan states that Abby is a liar. Chris declares that both Abby and Ben are liars. Chris also states that Dan is a liar as well. Who is what? Justify your answer.

21. The country of FarAwaynia is composed of several states: it also has several political parties. Once, a group of FarAwaynian politicians got together for a dinner. It is known that the group contained people from at least two different states and at least two different parties. Prove that there were at least two politicians at the dinner that both represented different states and belonged to different parties.

22. The weight of a gold bar is two-thirds of itself plus 5 pounds. What is the weight of the gold bar?

23. A koala bear starts climbing up a eucalyptus tree that is 20 meters high. He starts at the bottom, climbing 5 meters up every day and sliding down 4 meters every night. How many days and nights will it take the koala to reach the top of the tree?