# EXPECTED VALUE AND GAMES OF CHANCE

In the ocean again?!?

In all of the games below, there is a octopus who likes to recruit young mathematicians to play his games, plus one player (you can imagine it's you). Our currency will be sand dollars; assume all of the sand dollars have the same value.

1. Let's continue with our game from the end of last class. The octopus has six treasure chests, and each one has a different (integer) number of sand dollars inside, from 1 to 6. They are all mixed up, and you don't know which one is which. If you buy a ticket to this game, you can choose one treasure box and open it to see how many sand dollars are inside. Then you have a choice – you can keep that treasure chest OR you can set it aside, pay one extra sand dollar, and choose a different chest to keep. If you swap, you keep the second box. When should you swap and when should you keep the first box?

| which box | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| value if you keep this one | | | | | | |
| expected value if you swap* | | | | | | |

* Don't forget to subtract the sand dollar you must pay to have the chance to swap.

What is the fair price of a ticket for this game?

2. Homework challenge – suppose you have the same game as above, but you are permitted to swap your box up to two times. (Each swap costs one sand dollar.) What is the fair price of a ticket now?

3. Next game! The octopus has a deck of ten cards – half have spiny lobsters on them (numbered 1 through 5) and half have moon jellies (numbered 1 through 5). The octopus will shuffle and then deal 3 cards to you. If you are dealt exactly $N$ lobster cards, then you will win $2^N$ sand dollars. What is the fair price of this game? (Hint: do you remember permutations and combinations? Might be helpful here!)

4. Next game! The octopus has a deck of ten cards – half have spiny lobsters on them (numbered 1 through 5) and half have moon jellies (numbered 1 through 5). The octopus will shuffle and then deal the cards one at a time. If you see three or more moon jellies in a row, you win ten sand dollars. If not, the octopus wins, and you don't get a prize. What is the fair price to play this game?

5. Next game! The octopus has a deck of ten cards – half have spiny lobsters on them (numbered 1 through 5) and half have moon jellies (numbered 1 through 5). The octopus will shuffle and deal you three cards. If you have a pair (i.e. two cards with the same number on them), your prize is ten times the number on your pair, in sand dollars. What is the fair price for this game?

6. Next game! This one is pretty famous, so maybe you've heard of it before. The octopus is the host of a game show, and you are the contestant. You are given the choice of three doors: Behind one door is a new submarine; behind the others, worms. (Of course, since you love ocean adventures, you would strongly prefer to win the submarine and you absolutely would not like any worms.) You pick a door, say No. 1, and the octopus, who knows what's behind the doors, opens another door, say No. 3, and shows you that it has a worm. He then says to you, "Would you like to switch to door No. 2?" Should you switch or keep your original choice?[1]

[1] This is called the Monty Hall problem, and it usually has a new car and goats instead of a new submarine and worms.