AN INTRODUCTION TO SPHERICAL GEOMETRY

A photographer goes out looking for bears. From her camp, she walks one mile due south, then one mile due east, where she photographs a bear. Finally, after walking one more mile, she is back at camp. What color was the bear?

1. A warmup question: in plane geometry, we need to have an idea of what a point is and what a line is before we can state our postulates. What will play the roles of points and lines when we study geometry on a sphere? Why?
2. How do you measure angles between two lines on a sphere?
3. What are the possible values for the measures of the angles between two lines or line segments on a sphere? Let's do measurements in radians, not degrees.
4. How would you measure the distance between two points on a sphere? Keep in mind you must stay on the surface of the sphere; no shortcuts through the interior.
5. In the plane, two lines may intersect or they may be parallel. In 3-d, we also have skew lines. What are the possibilities for the intersection of two lines on a sphere?
6. If a circle is all points equidistant from a given point, what do spherical circles look like? Remember to measure distances in the spherical way, staying on the surface of the sphere.
7. In spherical geometry, could a line also be a circle?
8. Before we proceed, let's flip to the back. We will consider Euclid's five postulates for plane geometry and see how to modify them for spherical geometry. Can you imagine tools which would play the role of our compass and straightedge in the plane?
9. What is the surface area of a sphere? (We won't prove this today, but some of you will know it from your previous geometry studies.)
10. In the plane, a "biangle" (two-sided polygon) is not a sensible object, but on the sphere it is. Both sides will be spherical lines, i.e. great circles. (Biangles are also called lunes or digons on some places.) How could we find the area of a biangle?
11. Let's consider a triangle on a sphere, a three-sided polygon whose sides are all lines/great circles. Now things get extra crazy – do the angles of such a triangle still add up to $180°$, i.e. π radians? NOPE! What happens instead?
12. This question involves some fancy counting, plus some of our previous answers. Given the three angles of a spherical triangle, can we find the area? (An amazing observation – unlike in plane geometry, we don't need to know the side lengths of our triangles!)
13. On a sphere of radius R, just how large can the area of a triangle be? How small?
14. In plane geometry, two triangles are similar if they have two (and therefore three) congruent angles. Is this still true in spherical geometry?
15. Is there such a thing as a spherical rectangle (i.e. four-sided polygon with all right angles)? Describe one or explain why it's impossible.
16. What about a regular quadrilateral on a sphere – can you have four congruent sides and four congruent angles? Regular pentagon, hexagon, etc.?

Euclid's Postulates for plane geometry	Spherical Geometry Analogues