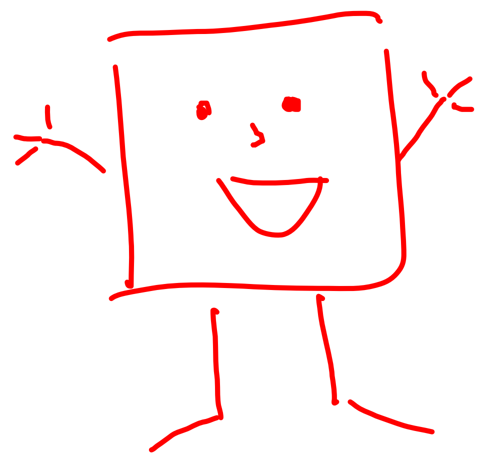
Ken Ken Challenges

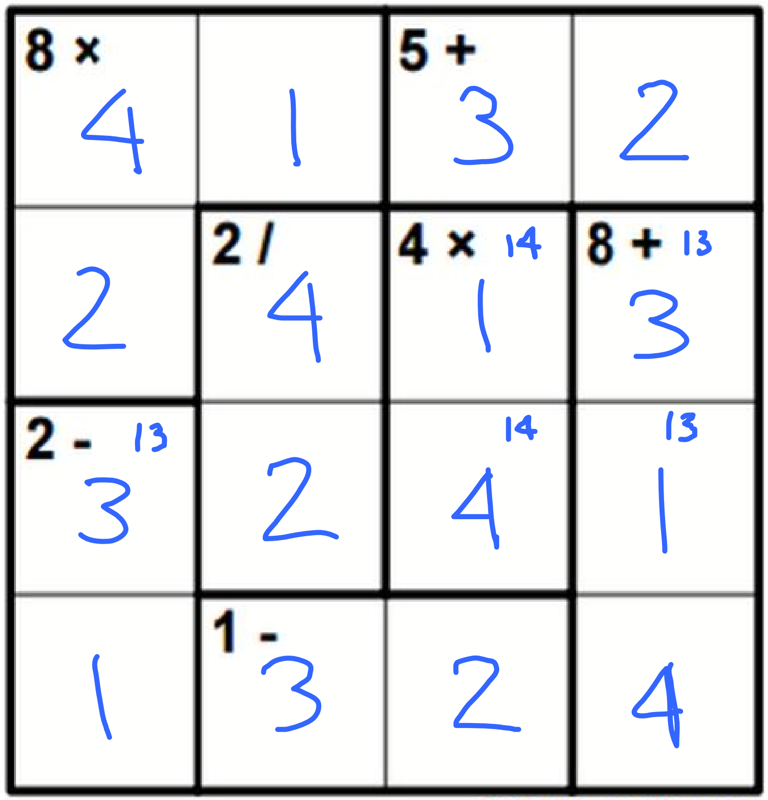
with Kelli Talaska



Based on notes by
Dr. Harold Reiter

THE RULES

In the standard $n \times n$ KenKen puzzle, the numbers in each heavily outlined set of squares, called cages, must combine (in any order) to produce the target number in the top corner of the cage using the mathematical operation indicated. Each of the numbers 1 to n must appear in each (horizontal) row and each (vertical) column. A number can be repeated within a cage as long as it is not in the same row or column.



8× 142

2- : 1 and 3 or 2 and 4

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2 - 13 3	13 1	32 × 24 2	4
2 / 2	1 - 3	24 4	1
4	2	3 × 13 1	5 + 23 3
5 + 14 1	14 4	13 3	23 2

5+ : 1 and 4 or 2 and 3

1- : 1,2 / 2,3 / 3,4

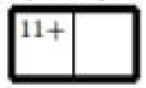
What techniques have we already used?

Process of elimination.

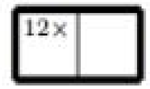
Keep track of possibilities.

Arithmetic

Parity and fault lines

A *fault line* is a heavy line that cuts entirely through the puzzle. Fault lines often provide the opportunity to use parity or other ideas because they cut the puzzle into a smaller puzzle of manageable size. *Parity* refers to evenness and oddness of a cage. Specifically, the parity of a cage C is *even (odd)* if the sum of the entries of the cage is an even (odd) number. For example, 

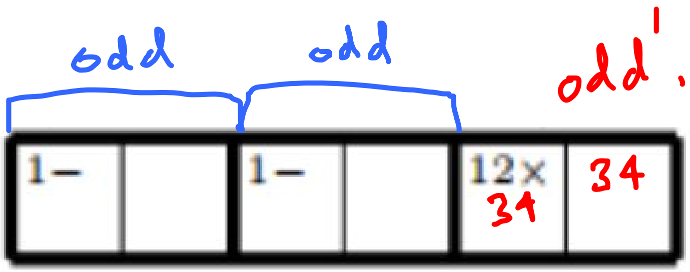
is an odd cage the sum of the entries is 11, which is an odd number. Some two-cell cages have determined parity even though the candidates are not determined. For example, 

is an even cage because the entries are either both even or both odd. On the other hand there are two-cell cages that can be either even or odd. For example, 

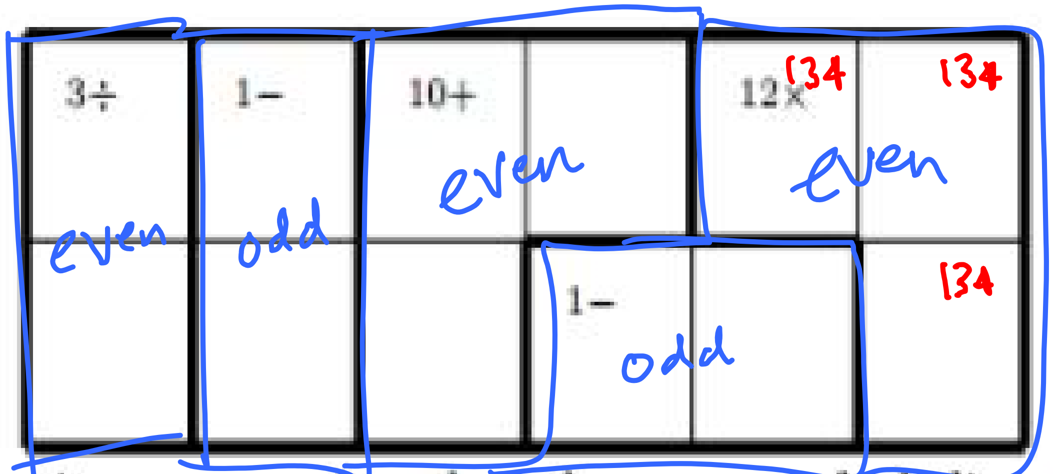
has two pairs of candidates, $\{2, 6\}$ and $\{3, 4\}$.

So how can we use parity to make progress towards a solution? Consider the row from a 6×6 KenKen:

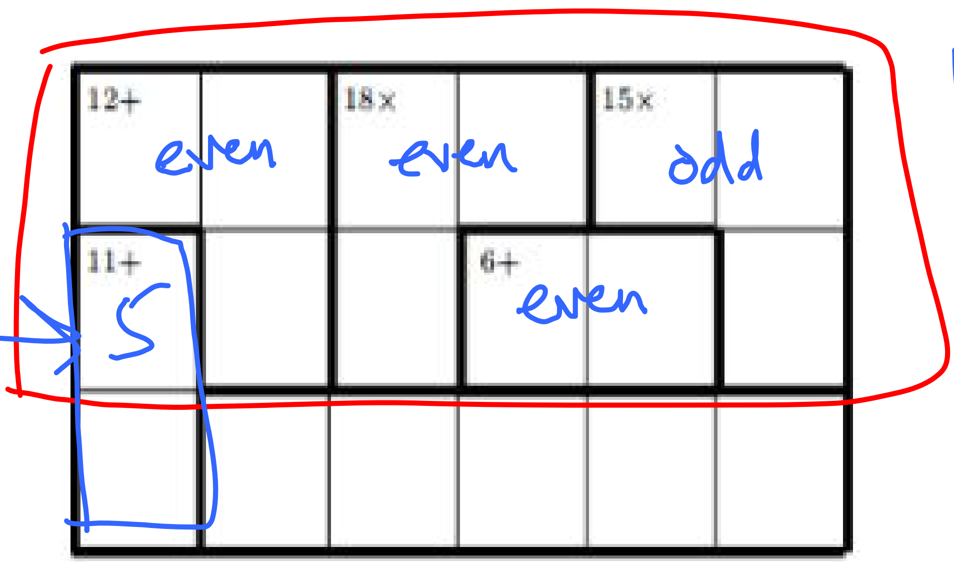
whole row total
1+2+3+4+5+6
= 21



consider the two-row KenKen fragment below.

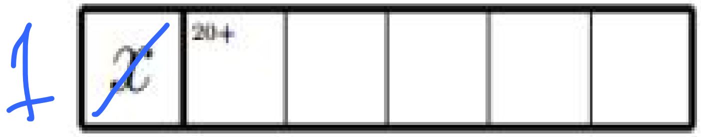


Of course we can sometimes use parity when there are no fault lines. Consider the puzzle part below:

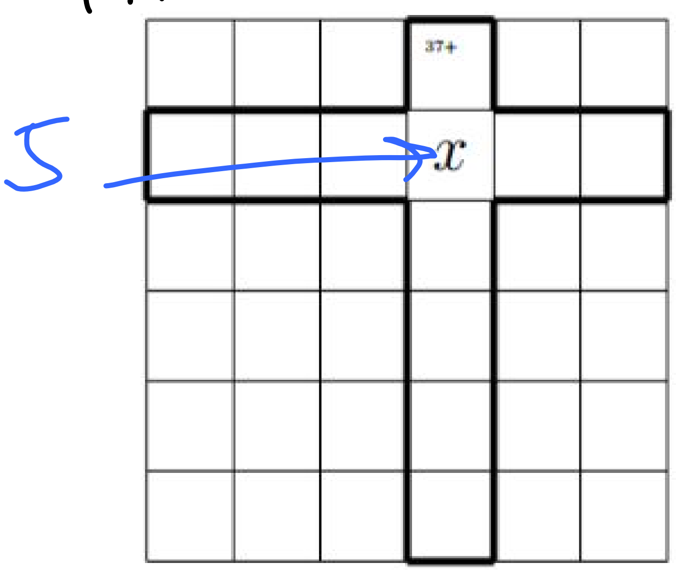


Counting (and knowing your triangular numbers).

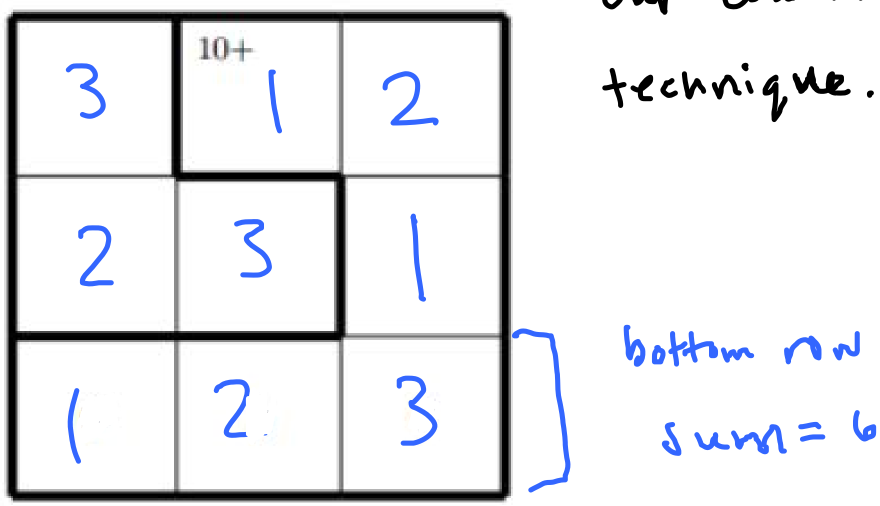
Consider the 6×6 KenKen fragment below. Find the digit that goes in the cell with the x. That is, find the value of x.



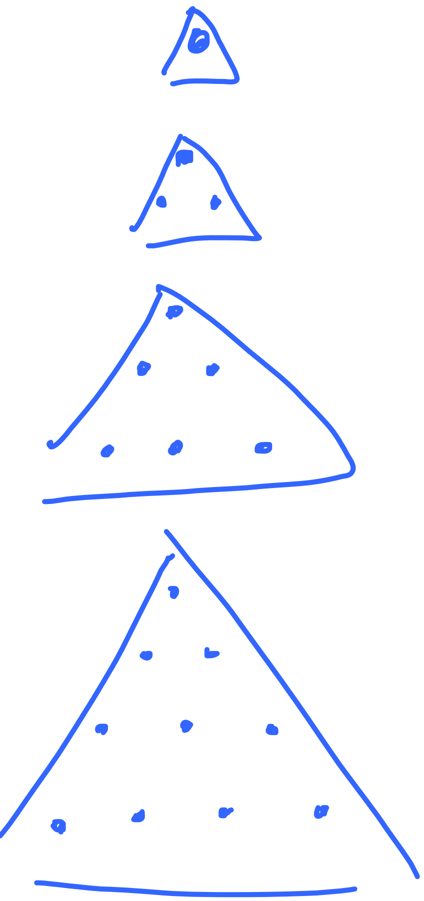
Find x here too! 37+



You can totally solve this 3x3 using our counting technique.



Sidebar: Triangular Numbers and Factorials



How many dots

$1 = 1$

$1+2 = 3$

$1+2+3 = 6$

$1+2+3+4 = 10$

Skip 5

$1+2+3+4+5+6 = 21$

$1! = 1$

$2! = 1 \cdot 2 = 2$

$3! = 1 \cdot 2 \cdot 3 = 6$

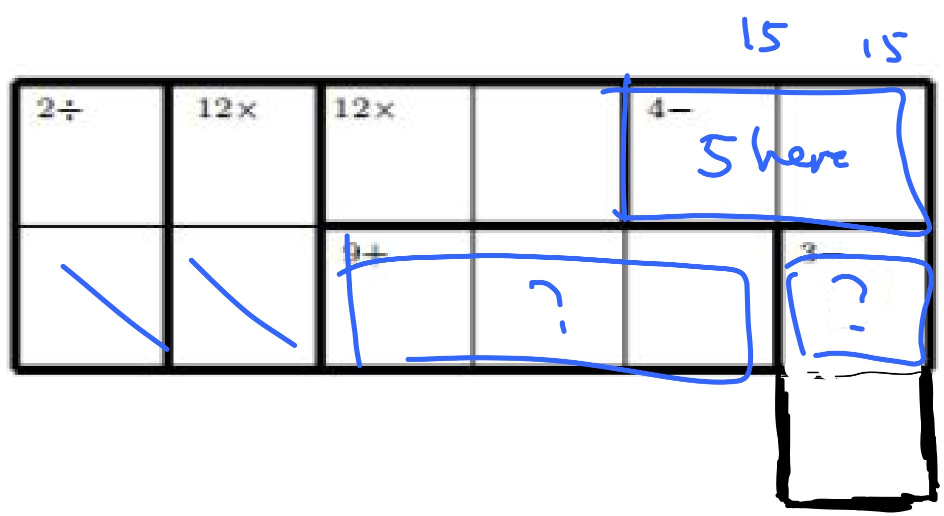
$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

$6! = 720$

The 5's must go somewhere!

(this one is almost too obvious to mention, but it's frequently helpful.)

In this fragment of a 6x6 puzzle, which cages have 5's?



Stacked Cages

Some puzzles have two or more cages confined to a single line (a row or a column). In this case, we call the cages *stacked*, and we can often take advantage of this situation. Consider the fragment below.

24×			2−		x

See what inferences you can draw in each of the following cases.

1.

2÷		2÷		6+	

2.

2÷		2÷		1−	

3.

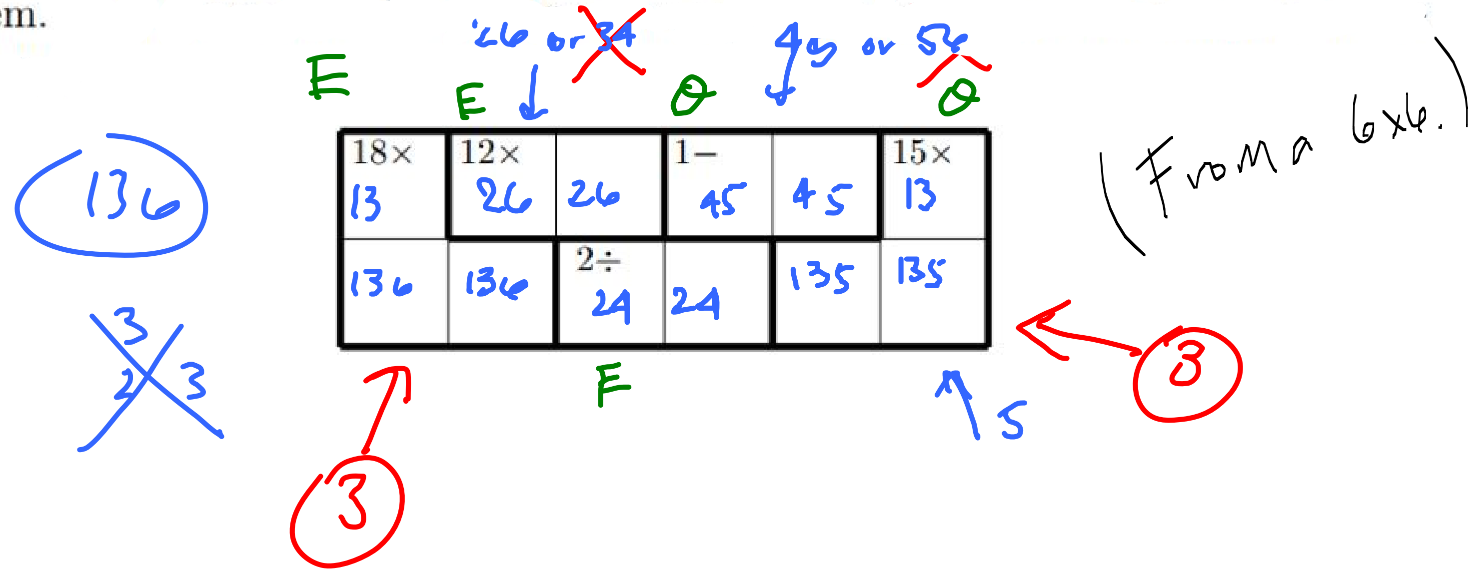
2−		2−		6×	

4.

2÷		3÷		$x-y$	$x-y$

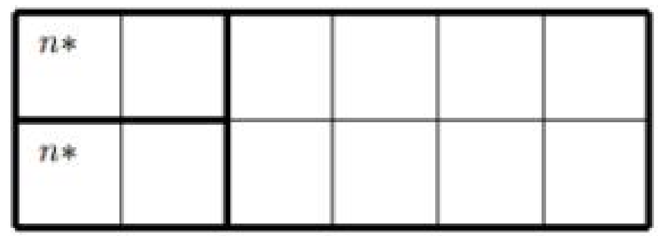
The X-wing strategy

The X-wing strategy refers to the fact that no k parallel lines can have more than k copies of a given symbol. In the sample case below, we use the fact that there are at most two 2's in the two rows, and then use parity and counting to finish the problem.

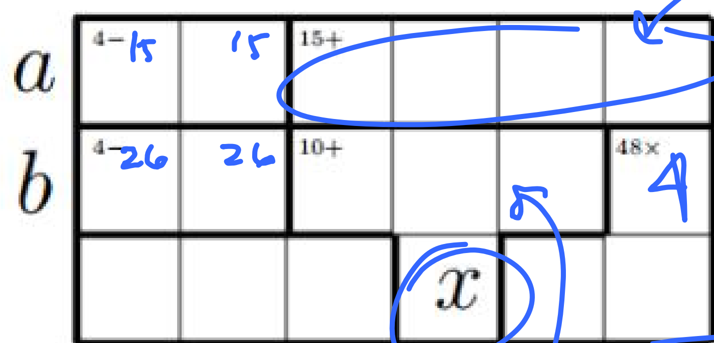


Parallel Cages

Suppose a two-cell cage [$n*$] appears in two parallel lines in the same position within the line. For example,



Then the required uniqueness of the solution implies that the two cages cannot be filled with the same two-element set. Consider the example below. Find the value of x.

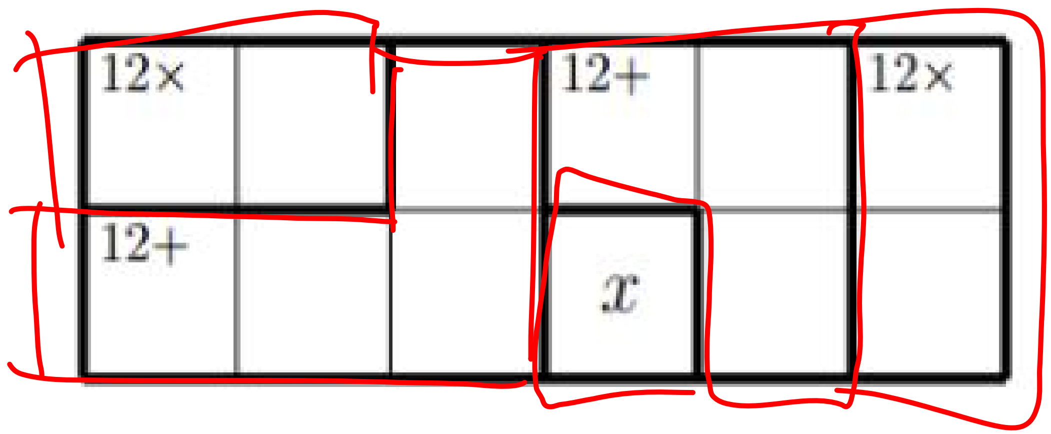


Orthogonal Cages

A simple example of orthogonality is shown below.

$x = 3$

12x : 6 2, 3 4

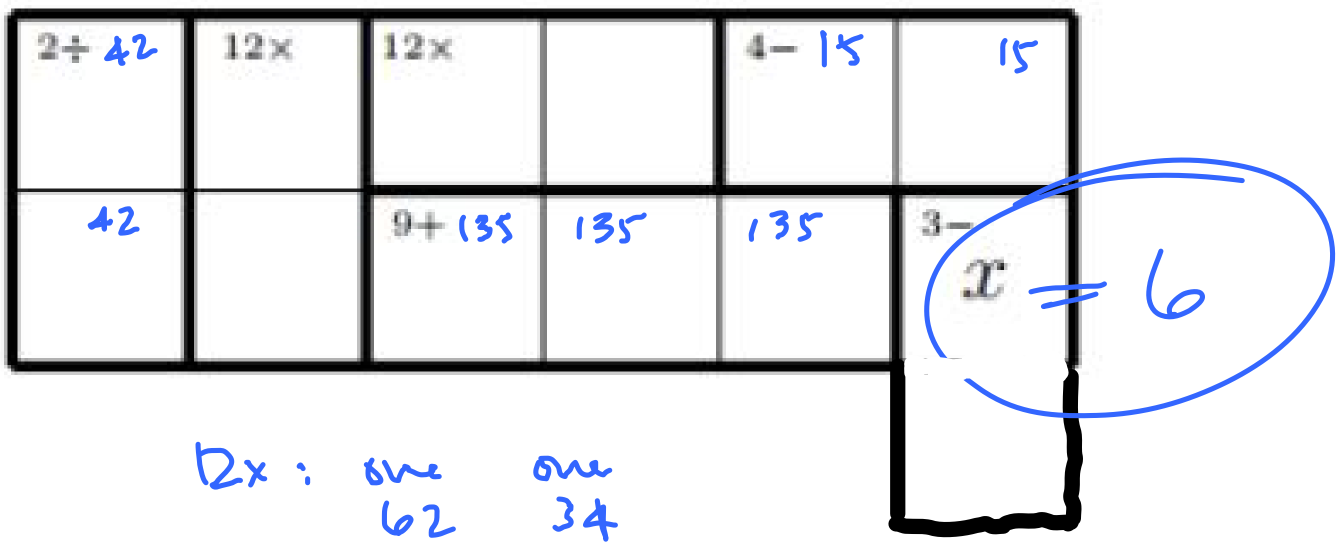


Find the value of x.

whole 2 row total = 42

so far 8 + 7 + 12 + 12 = 39

A Test! Can you find x?



12x : one 62, one 34

2÷ : ~~63~~, 42, ~~21~~

9+ : 531

As usual, part of a 6x6.

HW: complete ≥ 2 puzzles