9th Annual American Mathematics Contest 10

AMC 10
CONTEST B

THE MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.

2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.

3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.

4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.

5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** to complete the test.

8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

   Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 26th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 18, 2008 or Wednesday, April 2, 2008. More details about the AIME and other information are on the back page of this test booklet.

1. A basketball player made 5 baskets during a game. Each basket was worth either 2 or 3 points. How many different numbers could represent the total points scored by the player?

   (A) 2   **(B)** 3   **(C)** 4   **(D)** 5   **(E)** 6

2. A 4×4 block of calendar dates is shown. The order of the numbers in the second row is to be reversed. Then the order of the numbers in the fourth row is to be reversed. Finally, the numbers on each diagonal are to be added. What will be the positive difference between the two diagonal sums?

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

   (A) 2   **(B)** 4   **(C)** 6   **(D)** 8   **(E)** 10

3. Assume that x is a positive real number. Which is equivalent to $\sqrt[3]{x\sqrt{x}}$?

   (A) $x^{1/6}$ **(B)** $x^{1/4}$ **(C)** $x^{3/8}$ **(D)** $x^{1/2}$ **(E)** x

4. A semipro baseball league has teams with 21 players each. League rules state that a player must be paid at least \$15,000, and that the total of all players' salaries for each team cannot exceed \$700,000. What is the maximum possible salary, in dollars, for a single player?

   (A) 270,000   **(B)** 385,000   **(C)** 400,000   **(D)** 430,000   **(E)** 700,000

5. For real numbers a and b, define $a\$b = (a-b)^2$. What is $(x-y)^2\$(y-x)^2$?

   (A) 0   **(B)** $x^2 + y^2$ **(C)** $2x^2$ **(D)** $2y^2$ **(E)** $4xy$

6. Points B and C lie on $\overline{AD}$. The length of $\overline{AB}$ is 4 times the length of $\overline{BD}$, and the length of $\overline{AC}$ is 9 times the length of $\overline{CD}$. The length of $\overline{BC}$ is what fraction of the length of $\overline{AD}$?

   (A) $\dfrac{1}{36}$ **(B)** $\dfrac{1}{13}$ **(C)** $\dfrac{1}{10}$ **(D)** $\dfrac{5}{36}$ **(E)** $\dfrac{1}{5}$

7. An equilateral triangle of side length 10 is completely filled in by non-overlapping equilateral triangles of side length 1. How many small triangles are required?

   (A) 10   **(B)** 25   **(C)** 100   **(D)** 250   **(E)** 1000

8. A class collects \$50 to buy flowers for a classmate who is in the hospital. Roses cost \$3 each, and carnations cost \$2 each. No other flowers are to be used. How many different bouquets could be purchased for exactly \$50 ?

   (A) 1   **(B)** 7   **(C)** 9   **(D)** 16   **(E)** 17

9. A quadratic equation $ax^2 - 2ax + b = 0$ has two real solutions. What is the average of the solutions?

   (A) 1   **(B)** 2   **(C)** $\dfrac{b}{a}$ **(D)** $\dfrac{2b}{a}$ **(E)** $\sqrt{2a - b}$

10. Points A and B are on a circle of radius 5 and $AB = 6$. Point C is the midpoint of the minor arc AB. What is the length of the line segment AC?

    (A) $\sqrt{10}$ **(B)** $\dfrac{7}{2}$ **(C)** $\sqrt{14}$ **(D)** $\sqrt{15}$ **(E)** 4

11. Suppose that (u_n) is a sequence of real numbers satisfying $u_{n+2} = 2u_{n+1} + u_n$, and that $u_3 = 9$ and $u_6 = 128$. What is u_5?

    (A) 40   **(B)** 53   **(C)** 68   **(D)** 88   **(E)** 104

12. Postman Pete has a pedometer to count his steps. The pedometer records up to 99999 steps, then flips over to 00000 on the next step. Pete plans to determine his mileage for a year. On January 1 Pete sets the pedometer to 00000. During the year, the pedometer flips from 99999 to 00000 forty-four times. On December 31 the pedometer reads 50000. Pete takes 1800 steps per mile. Which of the following is closest to the number of miles Pete walked during the year?

    (A) 2500   **(B)** 3000   **(C)** 3500   **(D)** 4000   **(E)** 4500

13. For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008th term of the sequence?

    (A) 2008   **(B)** 4015   **(C)** 4016   **(D)** 4,030,056   **(E)** 4,032,064

14. Triangle OAB has $O = (0,0)$, $B = (5,0)$, and A in the first quadrant. In addition, $\angle ABO = 90°$ and $\angle AOB = 30°$. Suppose that $\overline{OA}$ is rotated $90°$ counterclockwise about O. What are the coordinates of the image of A?

    (A) $\left(-\dfrac{10}{3}\sqrt{3}, 5\right)$ **(B)** $\left(-\dfrac{5}{3}\sqrt{3}, 5\right)$ **(C)** $\left(\sqrt{3}, 5\right)$ **(D)** $\left(\dfrac{5}{3}\sqrt{3}, 5\right)$

    (E) $\left(\dfrac{10}{3}\sqrt{3}, 5\right)$

15. How many right triangles have integer leg lengths a and b and a hypotenuse of length $b + 1$, where $b < 100$?

    (A) 6   **(B)** 7   **(C)** 8   **(D)** 9   **(E)** 10

16. Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, the sum is 0.)

    (A) $\dfrac{3}{8}$ **(B)** $\dfrac{1}{2}$ **(C)** $\dfrac{43}{72}$ **(D)** $\dfrac{5}{8}$ **(E)** $\dfrac{2}{3}$

17. A poll shows that 70% of all voters approve of the mayor's work. On three separate occasions a pollster selects a voter at random. What is the probability that on exactly one of these three occasions the voter approves of the mayor's work?

   (A) 0.063   (B) 0.189   (C) 0.233   (D) 0.333   (E) 0.441

18. Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together, they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?

   (A) 500   (B) 900   (C) 950   (D) 1000   (E) 1900

19. A cylindrical tank with radius 4 feet and height 9 feet is lying on its side. The tank is filled with water to a depth of 2 feet. What is the volume of the water, in cubic feet?

   (A) $24\pi - 36\sqrt{2}$ (B) $24\pi - 24\sqrt{3}$ (C) $36\pi - 36\sqrt{3}$ (D) $36\pi - 24\sqrt{2}$
   (E) $48\pi - 36\sqrt{3}$

20. The faces of a cubical die are marked with the numbers 1, 2, 2, 3, 3, and 4. The faces of a second cubical die are marked with the numbers 1, 3, 4, 5, 6, and 8. Both dice are thrown. What is the probability that the sum of the two top numbers will be 5, 7, or 9?

   (A) $\dfrac{5}{18}$ (B) $\dfrac{7}{18}$ (C) $\dfrac{11}{18}$ (D) $\dfrac{3}{4}$ (E) $\dfrac{8}{9}$

21. Ten chairs are evenly spaced around a round table and numbered clockwise from 1 through 10. Five married couples are to sit in the chairs with men and women alternating, and no one is to sit either next to or directly across from his or her spouse. How many seating arrangements are possible?

   (A) 240   (B) 360   (C) 480   (D) 540   (E) 720

22. Three red beads, two white beads, and one blue bead are placed in a line in random order. What is the probability that no two neighboring beads are the same color?

   (A) $\dfrac{1}{12}$ (B) $\dfrac{1}{10}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{3}$ (E) $\dfrac{1}{2}$

23. A rectangular floor measures a feet by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

    (A) 1   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** 5

24. Quadrilateral $ABCD$ has $AB = BC = CD$, $\angle ABC = 70°$, and $\angle BCD = 170°$. What is the degree measure of $\angle BAD$?

    (A) 75   **(B)** 80   **(C)** 85   **(D)** 90   **(E)** 95

25. Michael walks at the rate of 5 feet per second on a long straight path. Trash pails are located every 200 feet along the path. A garbage truck travels at 10 feet per second in the same direction as Michael and stops for 30 seconds at each pail. As Michael passes a pail, he notices the truck ahead of him just leaving the next pail. How many times will Michael and the truck meet?

    (A) 4   **(B)** 5   **(C)** 6   **(D)** 7   **(E)** 8

2008

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DO NOT OPEN UNTIL
WEDNESDAY, February 27, 2008

**Administration On An Earlier Date Will Disqualify
Your School's Results**

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE February 27.** Nothing is needed from inside this package until February 27.

2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form found in the Teachers' Manual.

3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.

4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination during this period via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.*

The American Mathematics Competitions
are Sponsored by

2008 AIME

The 26th annual AIME will be held on Tuesday, March 18, 2008, with the alternate on Wednesday, April 2, 2008. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above, or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on April 29 and 30, 2008. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.unl.edu/amc.