# 3 Knights and Knaves

## A. THE ISLAND OF KNIGHTS AND KNAVES

There is a wide variety of puzzles about an island in which certain inhabitants called "knights" always tell the truth, and others called "knaves" always lie. It is assumed that every inhabitant of the island is either a knight or a knave. I shall start with a well-known puzzle of this type and then follow it with a variety of puzzles of my own.

### 26.

According to this old problem, three of the inhabitants—A, B, and C—were standing together in a garden. A stranger passed by and asked A, "Are you a knight or a knave?" A answered, but rather indistinctly, so the stranger could not make out what he said. The stranger than asked B, "What did A say?" B replied, "A said that he is a knave." At this point the third man, C, said, "Don't believe B; he is lying!"

The question is, what are B and C?

### 27.

When I came upon the above problem, it immediately



struck me that C did not really function in any essential way; he was sort of an appendage. That is to say, the moment B spoke, one could tell without C's testimony that B was lying (see solution). The following variant of the problem eliminates that feature.

Suppose the stranger, instead of asking A what he is, asked A, "How many knights are among you?" Again A answers indistinctly. So the stranger asks B, "What did A say? B replies, "A said that there is one knight among us." Then C says, "Don't believe B; he is lying!"

Now what are B and C?

### 28.

In this problem, there are only two people, A and B, each of whom is either a knight or a knave. A makes the following statement: "At least one of us is a knave."

What are A and B?

### 29.

Suppose A says, "Either I am a knave or B is a knight." What are A and B?

### 30.

Suppose A says, "Either I am a knave or else two plus two equals five." What would you conclude?

### 31.

Again we have three people, A, B, C, each of whom is either a knight or a knave. A and B make the following statements:

*A*: All of us are knaves.
*B*: Exactly one of us is a knight.

What are A, B, C?

## 32.

Suppose instead, A and B say the following:

*A*: All of us are knaves.
*B*: Exactly one of us is a knave.

Can it be determined what B is? Can it be determined what C is?

## 33.

Suppose A says, "I am a knave, but B isn't."

What are A and B?

## 34.

We again have three inhabitants, A, B, and C, each of whom is a knight or a knave. Two people are said to be of the *same type* if they are both knights or both knaves. A and B make the following statements:

*A*: B is a knave.
*B*: A and C are of the same type.

What is C?

## 35.

Again three people A, B, and C. A says "B and C are of the same type." Someone then asks C, "Are A and B of the same type?"

What does C answer?

## 36. An Adventure of Mine.

This is an unusual puzzle; moreover it is taken from real life. Once when I visited the island of knights and knaves, I



came across two of the inhabitants resting under a tree. I asked one of them, "Is either of you a knight?" He responded, and I knew the answer to my question.

What is the person to whom I addressed the question–is he a knight or a knave; And what is the other one? I can assure you, I have given you enough information to solve this problem.

## 37.

Suppose *you* visit the island of knights and knaves. You come across two of the inhabitants lazily lying in the sun. You ask one of them whether the other one is a knight, and you get a (yes-or-no) answer. Then you ask the second one whether the first one is a knight. You get a (yes-or-no) answer.

Are the two answers necessarily the same?

## 38. Edward or Edwin?

This time you come across just one inhabitant lazily lying in the sun. You remember that his first name is either Edwin or Edward, but you cannot remember which. So you ask him his first name and he answers "Edward."

What is his first name?

# B. KNIGHTS, KNAVES, AND NORMALS

An equally fascinating type of problem deals with three types of people: knights, who always tell the truth; knaves, who always lie; and normal people, who sometimes lie and sometimes tell the truth. Here are some puzzles of mine about knights, knaves, and normals.

## 39.

We are given three people, A,B,C, one of whom is a knight,

# 4 Alice in the Forest of Forgetfulness

## A. THE LION AND THE UNICORN

When Alice entered the Forest of Forgetfulness, she did not forget *everything*; only certain things. She often forgot her name, and the one thing she was most likely to forget was the day of the week. Now, the Lion and the Unicorn were frequent visitors to the forest. These two are strange creatures. The Lion lies on Mondays, Tuesdays, and Wednesdays and tells the truth on the other days of the week. The Unicorn, on the other hand, lies on Thursdays, Fridays, and Saturdays, but tells the truth on the other days of the week,

### 47.

One day Alice met the Lion and the Unicorn resting under a tree. They made the following statements:

*Lion* / Yesterday was one of my lying days.
*Unicorn* / Yesterday was one of my lying days too.

From these two statements, Alice (who was a very bright girl) was able to deduce the day of the week. What day was it?



### 48.

On another occasion Alice met the Lion alone. He made the following two statements:

(1) I lied yesterday.
(2) I will lie again two days after tomorrow.

What day of the week was it?

### 49.

On what days of the week is it possible for the Lion to make the following two statements:

(1) I lied yesterday.
(2) I will lie again tomorrow.

### 50.

On what days of the week is it possible for the Lion to make the following single statement: "I lied yesterday and I will lie again tomorrow." *Warning*! The answer is *not* the same as that of the preceding problem!

## B. TWEEDLEDUM AND TWEEDLEDEE

During one month the Lion and the Unicorn were absent from the Forest of Forgetfulness. They were elsewhere, busily fighting for the crown.

However, Tweedledum and Tweedledee were frequent visitors to the forest. Now, one of the two is like the Lion, lying on Mondays, Tuesdays, and Wednesdays and telling the truth on the other days of the week. The other one is like the Unicorn; he lies on Thursdays, Fridays, and Saturdays but tells the truth the other days of the week. Alice didn't know which one was like the Lion and which one was like the Unicorn. To make matters worse, the brothers looked so much alike, that Alice could not even tell them apart (except when they wore their embroidered collars, which they seldom did). Thus poor Alice found the situation most confusing indeed! Now, here are some of Alice's adventures with Tweedledum and Tweedledee.

## 51.

One day Alice met the brothers together and they made the following statements:

*First One* / I'm Tweedledum.
*Second One* / I'm Tweedledee.

Which one was really Tweedledum and which one was Tweedledee?

## 52.

On another day of that same week, the two brothers made the following statements:

*First One* / I'm Tweedledum.
*Second One* / If that's really true, then I'm Tweedledee!

Which was which?

## 53.

On another occasion, Alice met the two brothers, and asked one of them, "Do you lie on Sundays?" He replied "Yes." Then she asked the other one the same question. What did he answer?



## 54.

On another occasion, the brothers made the following statements:

*First One* / (1) I lie on Saturdays.
(2) I lie on Sundays.
*Second One* / I will lie tomorrow.

What day of the week was it?

## 55.

One day Alice came across just one of the brothers. He made the following statement: "I am lying today and I am Tweedledee."

Who was speaking?

## 56.

Suppose, instead, he had said: "I am lying today or I am Tweedledee." Would it have been possible to determine who it was?

## 57.

One day Alice came across both brothers. They made the following statements:

*First One* / If I'm Tweedledum then he's Tweedledee.
*Second One* / If he's Tweedledee then I'm Tweedledum.

Is it possible to determine who is who? Is it possible to determine the day of the week?

SOLUTIONS

This Book is Awesome!

# WHAT IS THE NAME OF THIS BOOK?

## The Riddle of Dracula and Other Logical Puzzles

RAYMOND M. SMULLYAN

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false since neither was a knight). So if A had answered "Yes" I would have had no way of knowing. But I told you that I *did* know after A's answer. Therefore A must have answered "No."

The reader can now easily see what A and the other—call him B—must be: If A were a knight, he couldn't have truthfully answered "No," so A is a knave. Since his answer "No" is false, then there is at least one knight present. Hence A is a knave and B is a knight.

### 37.

Yes, they are. If they are both knights, then they will both answer "Yes." If they are both knaves, then again they will both answer "Yes." If one is a knight and the other a knave, then the knight will answer "No," and the knave will also answer "No."

### 38.

I feel entitled, occasionally, to a little horseplay. The vital clue I gave you was that the man was lazily lying in the sun. From this it follows that he was lying in the sun. From this it follows that he was *lying*, hence he is a knave. So his name is Edwin.

### 39.

To begin with, A cannot be a knight, because a knight would never say that he is normal. So A is a knave or is normal.

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What are Mr. and Mrs. A?

## 45.

Suppose, instead, they had said:

> *Mr. A* / My wife is normal.
> *Mrs. A* / My husband is normal.

Would the answer have been different?

## 46.

This problem concerns two married couples on the island of Bahava, Mr. and Mrs. A, and Mr. and Mrs. B. They are being interviewed, and three of the four people give the following testimony:

> *Mr. A* / Mr. B is a knight.
> *Mrs. A* / My husband is right; Mr. B is a knight.
> *Mrs. B* / That's right. My husband is indeed a knight.

What are each of the four people, and which of the three statements are true?

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## 26.

It is impossible for either a knight or a knave to say, "I'm a knave," because a knight wouldn't make the false statement that he is a knave, and a knave wouldn't make the true statement that he is a knave. Therefore A never did say that he was a knave. So B lied when he said that A said that he was a knave. Hence B is a knave. Since C said that B was lying and B was indeed lying, then C spoke the truth, hence



is a knight. Thus B is a knave and C is a knight. (It is impossible to know what A is.)

## 27.

The answer is the same as that of the preceding problem, though the reasoning is a bit different.

The first thing to observe is that B and C must be of opposite types, since B contradicts C. So of these two, one is a knight and the other a knave. Now, if A were a knight, then there would be two knights present, hence A would not have lied and said there was only one. On the other hand, if A were a knave, then it would be true that there was exactly one knight present; but then A, being a knave, couldn't have made that true statement. Therefore A could not have said that there was one knight among them. So B falsely reported A's statement, and thus B is a knave and C is a knight.

## 28.

Suppose A were a knave. Then the statement "At least one of us is a knave" would be false (since knaves make false statements); hence they would both be knights. Thus, if A were a knave he would also have to be a knight, which is impossible. Therefore A is not a knave; he is a knight. Therefore his statement must be true, so at least one of them really is a knave. Since A is a knight, then B must be the knave. So A is a knight and B is a knave.

## 29.

This problem is a good introduction to the logic of disjunction. Given any two statements $p, q$, the statement "either $p$ or $q$" means that at least one (and possibly both) of the statements $p,q$ are true. If the statement "either $p$ or $q$" should be false, then *both* the statements $p,q$ are false. For

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We now know that A is a knave and that B is a knight. Since B is a knight, his statement is true, so there is exactly one knight among them. This knight must be B, hence C must be a knave. Thus the answer is that A is a knave, B is a knight, and C is a knave.

## 32.

It cannot be determined what B is, but it can be proved that C is a knight.

To begin with, A must be a knave for the same reasons as in the preceding problem; hence also there is at least one knight among them. Now, either B is a knight or a knave. Suppose he is a knight. Then it is true that exactly one of them is a knave. This only knave must be A, so C would be a knight. So if B is a knight, so is C. On the other hand, if B is a knave, then C must be a knight, since all three can't be knaves (as we have seen). So in either case, C must be a knight.

## 33.

To begin with, A can't be a knight or his statement would be true, in which case he would have to be a knave. Therefore A is a knave. Hence also his statement is false. If B were a knight, then A's statement would be true. Hence B is also a knave. So A,B are both knaves.

## 34.

Suppose A is a knight. Then his statement that B is a knave must be true, so B is then a knave. Hence B's statement that A and C are of the same type is false, so A and C are of different types. Hence C must be a knave (since A is a knight). Thus if A is a knight, then C is a knave.

On the other hand, suppose A is a knave. Then his statement that B is a knave is false, hence B is a knight.



Hence B's statement is true that A and C are of the same type. This means that C must be a knave (since A is).

We have shown that regardless of whether A is a knight or a knave, C must be a knave. Hence C is a knave.

## 35.

I'm afraid we can solve this problem only by analysis into cases.

*Case One:* A is a knight. Then B,C really are of the same type. If C is a knight, then B is also a knight, hence is of the same type as A, so C being truthful must answer "Yes." If C is a knave, then B is also a knave (since he is the same type as C), hence is of a different type than A. So C, being a knave, must lie and say "Yes."

*Case Two:* A is a knave. Then B,C are of different types. If C is a knight, then B is a knave, hence he is of the same type as A. So C, being a knight, must answer "Yes." If C is a knave, then B, being of a different type than C, is a knight, hence is of a different type than A. Then C, being a knave, must lie about A and C being of different types, so he will answer "Yes."

Thus in both cases, C answers "Yes."

## 36.

To solve this problem, you must use the information I gave you that after the speaker's response, I knew the true answer to my question.

Suppose the speaker—call him A—had answered "Yes." Could I have then known whether at least one of them was a knight? Certainly not. For it could be that A was a knight and truthfully answered "Yes" (which would be truthful, since at least one—namely A—was a knight), or it could be that both of them were knaves, in which case A would have falsely answered "Yes" (which would indeed be

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example, if I should say, "Either it is raining or it is snowing," then if my statement is incorrect, it is both false that it is raining and false that it is snowing.

This is the way "either/or" is used in logic, and is the way it will be used throughout this book. In daily life, it is sometimes used this way (allowing the possibility that both alternatives hold) and sometimes in the so-called "exclusive" sense—that one and only one of the conditions holds. As an example of the exclusive use, if I say; "I will marry Betty or I will marry Jane," it is understood that the two possibilities are mutually exclusive—that is, that I will not marry both girls. On the other hand, if a college catalogue states that an entering student is required to have had *either* a year of mathematics *or* a year of a foreign language, the college is certainly not going to exclude you if you had both! This is the "*inclusive*" use of "either/or" and is the one we will constantly employ.

Another important property of the disjunction relation "either this or that" is this. Consider the statement "*p* or *q*" (which is short for "either *p* or *q*"). Suppose the statement happens to be true. Then if *p* is false, *q* must be true (because at least one of them is true, so if *p* is false, *q* must be the true one). For example, suppose it is true that it is either raining or snowing, but it is false that it is raining. Then it must be true that it is snowing.

We apply these two principles as follows. A made a statement of the disjunctive type: "Either I am a knave or B is a knight." Suppose A is a knave. Then the above statement must be false. This means that it is neither true that A is a knave nor that B is a knight. So if A were a knave, then it would follow that he is not a knave—which would be a contradiction. Therefore A must be a knight.

We have thus established that A is a knight. Therefore his statement is true that at least one of the possibilities holds: (1) A is a knave; (2) B is a knight. Since possibility (1) is false (since A is a knight) then possibility (2) must be the correct one, i.e., B is a knight. Hence A,B, are both knights.



## 30.

The only valid conclusion is that the author of this problem is not a knight. The fact is that neither a knight nor a knave could possibly make such a statement. If A were a knight, then the statement that either A is a knave or that two plus two equals five would be false, since it is neither the case that A is a knave nor that two plus two equals five. Thus A, a knight, would have made a false statement, which is impossible. On the other hand, if A were a knave, then the statement that either A is a knave or that two plus two equals five would be true, since the first clause that A is a knave is true. Thus A, a knave, would have made a true statement, which is equally impossible.

Therefore the conditions of the problem are contradictory (just like the problem of the irresistible cannonball and the immovable post). Therefore, I, the author of the problem, was either mistaken or lying. I can assure you I wasn't mistaken. Hence it follows that I am not a knight.

For the sake of the records, I would like to testify that I have told the truth at least once in my life, hence I am not a knave either.

## 31.

To begin with, A must be a knave, for if he were a knight, then it would be true that all three are knaves and hence that A too is a knave. If A were a knight he would have to be a knave, which is impossible. So A is a knave. Hence his statement was false, so in fact there is at least one knight among them.

Now, suppose B were a knave. Then A and B would both be knaves, so C would be a knight (since there is at least one knight among them). This would mean that there was exactly one knight among them, hence B's statement would be true. We would thus have the impossibility of a knave making a true statement. Therefore B must be a knight.

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versions of what really happened. How come four versions? Well, to tell you the truth, I didn't invent these stories myself; I heard them all from the mouth of the Jabberwocky. Now, the conversation between Alice and Humpty Dumpty really happened: Alice told me this herself, and Alice is always truthful. But the four versions of what happened after that were all told to me by the Jabberwocky. Now, I know that the Jabberwocky lies on the same days as the Lion (Monday, Tuesday, Wednesday) and he told me these stories on four consecutive weekdays. (I know they were weekdays, because I am lazy and sleep all day Saturdays and Sundays.) They were told to me in the same order as I recounted them.

From this information, the reader should have no difficulty in ascertaining whether Tweedledoo really exists or whether Humpty Dumpty was lying. Does Alice know whether Tweedledoo exists?

## SOLUTIONS

### 47.

The only days the Lion can say "I lied yesterday" are Mondays and Thursdays. The only days the Unicorn can say "I lied yesterday" are Thursdays and Sundays. Therefore the only day they can both say that is on Thursday.

### 48.

The lion's first statement implies that it is Monday or Thursday. The second statement implies that it is not Thursday. Hence it is Monday.

### 49.

On no day of the week is this possible! Only on Mondays and Thursdays could he make the first statement; only on



Wednesdays and Sundays could he make the second. So there is no day he could say both.

### 50.

This is a very different situation! It well illustrates the difference between making two statements separately and making one statement which is the conjunction of the two. Indeed, given any two statements *X*, *Y*, if the single statement "*X* and *Y*" is true, then it of course follows that *X*, *Y* are true separately; but if the conjunction "*X* and *Y*" is false, it only follows that at least one of them is false.

Now, the only day of the week it could be true that the Lion lied yesterday and will lie again tomorrow is Tuesday (this is the one and only day which occurs between two of the Lion's lying days). So the day the Lion said that couldn't be Tuesday, for on Tuesdays that statement is true, but the Lion doesn't make true statements on Tuesdays. Therefore it is not Tuesday, hence the Lion's statement is false, so the Lion is lying. Therefore the day must be either Monday or Wednesday.

### 51.

If the first statement is true, then the first one really is Tweedledum, hence the second one is Tweedledee and the second statement is also true. If the first statement is false, then the first one is actually Tweedledee and the second one is Tweedledum, and hence the second statement is also false. Therefore either both statements are true or both statements are false. They can't both be false, since the brothers never lie on the same day. Therefore both statements must be true. So the first one is Tweedledum and the second one is Tweedledee. Also, the day of the encounter must be Sunday.

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## 52.

This is a horse of a very different color! The second one's statement is certainly true. Now, we are given that the day of the week is different from that of the last problem, so it is a weekday. Therefore it cannot be that both statements are true, so the first one must be false. Therefore the first one is Tweedledee and the second is Tweedledum.

## 53.

The first answer was clearly a lie, hence the event must have taken place on a weekday. Therefore the other one must have answered truthfully and said "No."

## 54.

Statement (2) of the first one is clearly false, hence statement (1) is false too (since it is uttered on the same day). Therefore the first one does not lie on Saturdays, so the second one lies on Saturdays. The second one is telling the truth on this day (since the first one is lying), so it is now Monday, Tuesday, or Wednesday. The only one of these days in which it is true that he will lie tomorrow is Wednesday. So the day is Wednesday.

## 55.

His statement is certainly false (for if it were true, then he would be lying today, which is a contradiction). Therefore at least one of the two clauses "I am lying today," "I am Tweedledee" must be false. The first clause ("I am lying today") is true, therefore the second clause must be false. So he is Tweedledum.

## 56.

Yes it would. If he were lying today, then the first clause of the disjunction would be true, hence the whole statement would be true, which is a contradiction. Therefore he is telling the truth today. So his statement is true: either he is lying today or he is Tweedledee. Since he is not lying today, then he is Tweedledee.

## 57.

Both statements are obviously true, so it is a Sunday. It is not possible to determine who is who.

## 58.

To begin with, it is impossible on a Sunday for either brother to lie and say that it is not Sunday. Therefore today cannot be Sunday. So the first one is telling the truth, and (since it is not Sunday), the second one is therefore lying today. The second one says today is Monday, but he is lying, so it is not Monday either.

Now, the second one has also told the lie that the Lion lied yesterday, hence yesterday was really one of the Lion's truthful days. This means that yesterday was Thursday, Friday, Saturday, or Sunday, so today is Friday, Saturday, Sunday, or Monday. We have already ruled out Sunday and Monday, so today must be Friday or Saturday.

Next we observe that tomorrow is one of Tweedledee's lying days (since the first one, who is speaking the truth, said so). Therefore today cannot be Saturday. Hence today is Friday.

From this it further follows that Tweedledee lies on Saturdays, hence he is like the Unicorn. Also, the first one is telling the truth today, which is a Friday, hence he is Tweedledum. This proves everything.

## 59.

Suppose the first one told the truth. Then the rattle belongs to Tweedledee. The second speaker must be lying (since it is not Sunday), hence his name is not really Tweedledee; it