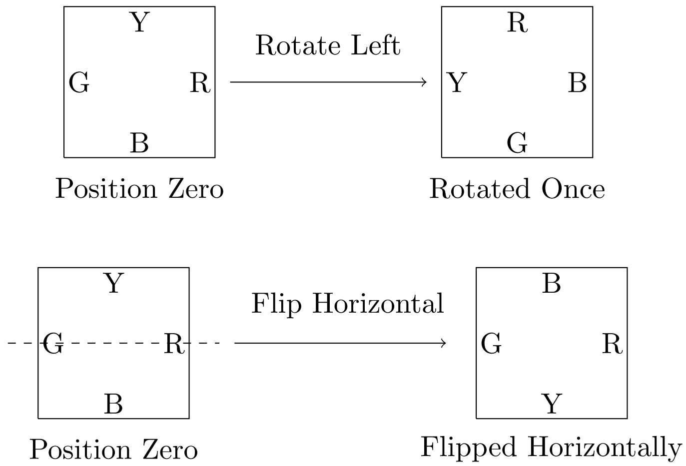
SYMMETRY GROUPS (PART 2)

Question 1. Write the addition table for the symmetry group of an equilateral triangle.

Question 2. Write the addition table for the symmetry group of a square. Use a model square and color the four edges. Here are some examples of symmetries of the square:



Challenge 1. How many symmetries are there of a regular pentagon? Can you write down the group table without using a model?

Challenge 2. Describe the addition table of a regular $n-$gon.

In the next few exercises we explore some properties of the above groups. We will now write D_3 for the group of symmetries of the equilateral triangle, and D_4 for the square.

Definition: If A is a symmetry, B is called the **inverse** of A if $A + B$ is equal to the 'do nothing' symmetry.

Question 3. Find the inverses of each symmetry in D_3 and D_4.

Definition: A **subgroup** of a group is some collection of symmetries such that when you add two of the symmetries, the resulting symmetry is still part of the collection.

Question 4. If a subgroup of D_3 contains R, which other elements must it also contain?

Question 5. If a subgroup of D_3 contains FV, which other elements must it also contain?

Question 6. If a subgroup of D_3 contains R *and* FV, which other elements must it also contain?

Question 7. List all subgroups of D_3.

Question 8. List three subgroups of D_4. At least one must contain a rotation, and at least one must contain a reflection.

Challenge 3. List all subgroups of D_4.

Definition. The **order** of a symmetry A is the number of times you need to combine A with itself in order to get the 'Do Nothing' symmetry. (The 'Do Nothing' symmetry has order one).

Question 9. Find the order of every element in D_3 and D_4.

Now we will investigate the symmetry group of a cube! To investigate this group we will use dice. In order to write down a symmetry, you need to be able to write down a position of a die.

There are six sides: Front (F), Back (B), Left (L), Right (R), Top (T), Bottom (Bo). So, for example you could write down a position of the die:

F: 1
B: 6
L: 2
R: 5
T: 3
Bo: 4

And you could write a symmetry:

$$\begin{array}{lcl} F:1 & & F:2 \\ B:6 & & B:5 \\ L:2 & \longrightarrow & L:6 \\ R:5 & RT & R:1 \\ T:3 & & T:3 \\ Bo:4 & & Bo:4 \end{array}$$

RT stands for "Rotate Top", which means rotate 90 degrees counterclockwise about the top face.

Question 10. Experiment with the die. Write down as many symmetries as you can. Rotate around other faces. Try rotating around edges or vertices. Write down the order of each symmetry you find.

Question 11. Write down two subgroups of the cube.

Question 12. Find a pair of symmetries that do not commute. That means, two symmetries A and B such that $A + B$ and $B + A$ are different symmetries.

Challenge 4. How many symmetries of the cube are there?

Question 13. There are three symmetries of the cube which involve rotating the cube 180 degrees around a pair of opposite faces. Show that those three symmetries, along with the 'Do Nothing' symmetry, are a subgroup of the symmetry group of a cube, and write down the addition table.