7. Circumcenter and Circumcircle

Ultimate Problem 1. Three villages *Abra*, *Cada*, and *Bra* would like to build a school. Where should they locate the school so that the children from all villages will walk the same distance to school?

Cada

Abra

Bra

Hint 1. What would the answer be if we had only two of the villages, say, Abra and Bra? Where should they build the school? Draw the answer on the picture.

Hint 2. What would the answer be if we had only the villages Bra and Cada? Where should they build the school? Draw the answer on the same picture.

Hint 3. Which point is *equidistant* from all three villages? How did you find it?

Follow-up Problem 2. Let *O* be the point where the school should be built in the previous problem. Draw a circle with center *O* that passes through village Abra. Will the circle also pass through villages Bra and Cada? Why?

Definition 1. A circle which passes through the three vertices of a triangle is called the *circumcircle* of the triangle. The center of the circle is called the *circumcenter* of the triangle.

Theorem 1. To find the circumcenter of triangle *ABC*, draw the perpendicular bisectors of two of the sides. Where they intersect will be the circumcenter of triangle *ABC*.

Question 1. What happened to the perpendicular bisector of the third side? Will it also pass through the circumcenter?

Problem 3. Draw three more triangles on your graph paper:

a) An *obtuse* triangle *EFG*.
b) A *right* triangle *HIJ*.
c) An *equilateral* triangle *KLM*.

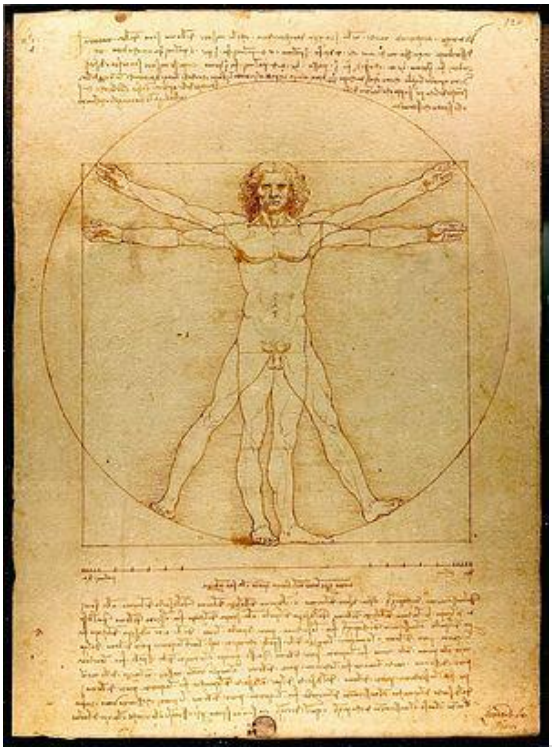
For each triangle, find its circumcenter and draw its circumcircle.

Problem 4. Let's reverse the situation. On your graph paper, draw first a circle k with center O. Can you quickly find a triangle ABC whose circumcircle is k? Can you find another triangle EFG whose circumcircle is also k?

Definition 2. When all vertices of a triangle lie on a circle we say that:

a) The triangle is *inscribed* in the circle.
b) The circle is *circumscribed* about the triangle.

Question 2. Which quadrilateral, do you think, we will call *inscribed* in a circle k? How about an *inscribed* n-gon in a circle k? On your graph paper, draw a picture of an inscribed quadrilateral in a circle k_1 and of an inscribed pentagon in another circle k_2.



Question 3. The famous drawing *Vitruvian Man* (1485) by *Leonardo da Vinci* depicts a man in two superimposed positions with his arms and legs apart. In which figure(s) is the person *inscribed*?

If you connect several well-chosen points on the person, what inscribed in the circle polygons can you obtain: an inscribed triangle, an inscribed quadrilateral, an inscribed pentagon, or an inscribed hexagon?

Historical Facts. The drawing is based on the correlations of ideal body proportions with geometry described by the ancient Roman architect *Vitruvius* in Book III of his treatise *De Architectura*.

Vitruvius described the human figure as being the principal source of proportion among the *Classical orders of architecture*. Vitruvius determined that the ideal body should be eight heads high. Moreover, when arms are stretched horizontally, the person should be inscribable in a square, and when the legs and arms make certain angles, the person should be inscribable in a circle. Find these angles with a protractor.

Problem 5. On your graph paper, draw a circle using a round cup or a something similar (but NOT compasses). Describe a way to find its center and perform your algorithm. Check in the end that indeed you found the center on the circle.

Hint 4. Draw an inscribed triangle in the circle. Forget temporarily about the circle and find the circumcenter of your triangle. Will this be the center of the circle?

Question 4. For a triangle *ABC*, how many circles are *circumscribed* about it? How many *circumcenters* does the triangle have?

Conversely, given a circle, how many triangles are *inscribed* in it? How many *centers* does the circle have?

Theorem 2. Every triangle has a unique circumcircle and a *unique* circumcenter. Every circle has a *unique* center.

But there are *infinitely many* triangles inscribed in every circle.

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RECAP 1: Check Your Understanding

Check the correct answers. Explain your choice and provide details.

1. A *circle* is:
   - I. A figure with infinitely many lines of symmetry.
   - II. The points in the plane that are equidistant from a given point, called the center of the circle.
   - III. What you will obtain if you cut a sphere through its equator.
   - IV. A figure which looks the same no matter which part of it you look at: if you chop up a small piece in one part, it will fit perfectly in any other part of the figure. (Such figures are called *self-congruent*.)
2. Any *three* points:
   - I. Form a triangle.
   - II. Form a line.
   - III. Determine a circle that passes through them.
   - IV. Not necessarily any of the above.
3. A *unique* circumcircle means:
   - I. A special circumcircle.
   - II. A non-existing circumcircle.
   - III. Exactly one circumcircle.
   - IV. More than one circumcircle.
4. Finding the exact *center* of a circle:
   - I. Is impossible.
   - II. Can be accomplished by successively approximating the location of the center and getting closer and closer.
   - III. Is an Euclidean construction.
   - IV. Can be done by intersecting two perpendicular bisectors.
5. A *square* is:
   - I. What you call someone who is boring.
   - II. A quadrilateral with four right angles.
   - III. A parallelogram that is a trapezoid.
   - IV. A rhombus with a right angle.
   - V. A rectangle with two equal adjacent sides.
   - VI. A quadrilateral whose diagonals are perpendicular, bisects each other, and have the same length.
   - VII. A polygon with several lines of symmetry.
6. The *Vitruvian Man* is:
   - I. A man with four legs and four arms.
   - II. What everyone should look like.
   - III. An artistic model of the ideal proportions the human body should have according to Vitruvius.
   - IV. A person simultaneously inscribed in a circle and in a square.