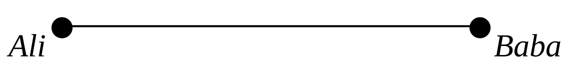
6. Perpendicular Bisector

Problem 1. Suppose you have two towns: *Ali* and *Baba*. They want to build a railroad in such a way that no matter where they place a railroad station along it, the people from the two towns will walk the *same distance* to the railroad station. Where should they build the railroad?



Hint 1. Find one good place for a station C_1 that is at equal distances from the two towns. Using your compasses might be helpful. Find now another such place for a station C_2, and then several more places.

Hint 2. Connect the places for the stations you found. What figure do they form? How does this figure relate to segment *AB* between the two towns? Is there a place for such a station *on* the segment *AB*? Which point is that: do you know a name for it?

Definition 1. A point *C* that is at the same distances from *A* and from *B* is called *equidistant from A and B*, i.e., *CA*=*CB*.

For example, all stations *C* that you found in the previous problem are *equidistant* from the towns Ali and Baba.

Theorem 1. Given a segment *AB*, the points that are equidistant from *A* and *B* form a line *l*. This line *l* has two properties:

a) Line *l* passes through the midpoint *M* of *AB*.
b) Line *l* is perpendicular to *AB*.

Definition 2. A line that passes through the midpoint of a segment and is perpendicular to the segment is called its *perpendicular bisector*.

Corollary 1. The fairest railroad for the two towns Ali and Baba turns out to be the *perpendicular bisector* of the segment *AB*.

Problem 2. Now check the opposite: pick any point *P* on the proposed railroad (or on the perpendicular bisector of *AB*), connect it to the two towns and verify that it is equidistant from them: is *PA* = *PB*?

Pick another point *Q* on the railroad and check again: is *QA* = *QB*?

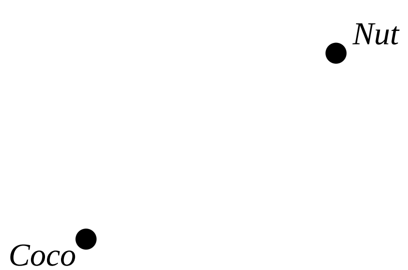
Question 1. Have we constructed before perpendicular bisectors, without realizing it? What were we trying to do before, what was the construction, and how did we do it?

Corollary 2. To find the *perpendicular bisector* of a segment *AB* we can:

- construct a *rhombus* with diagonal *AB.*

The other diagonal of the rhombus will be the perpendicular bisector of *AB*.

Problem 3. The houses of two kids *Coco* and *Nut* are drawn below. Find where they should build a fence between the houses so that no matter to which point on the fence the two kids run, they will always run the same distance.



Problem 4. In province *Faraway*, the *Water* purification center and the *Melon* patch are at the end of the universe, as shown on the next picture. Help them design a highway so that carrying water and carrying melons anywhere to the highway will be the same distance. Warning: the paper on the right ends (it is the end of the universe, after all! ☺), so you cannot use that part for your construction!

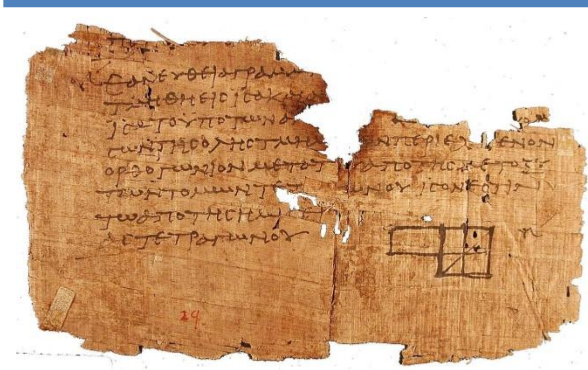
Hint 3. Do you always need to construct a *rhombus*, or could you get by constructing any two *isosceles* triangles? Should the isosceles triangles be on different sides of the segment *WM* or could they be drawn only on one side of the segment?

Corollary 3. To construct the *perpendicular bisector* of a segment *AB* it suffices to:

- construct an isosceles triangle *ABC* with base *AB*;
- construct another isosceles triangle *ABD*, also with base *AB;*
- connect the two vertices *C* and *D* of the isosceles triangles.

The line *CD* will be the perpendicular bisector of segment *AB*.

Historical facts. The perpendicular bisector appears in Propositions 1 and 3 in Book III of Euclid's *Elements*. (A fragment of the original manuscript is included below.)



RECAP 1: New Vocabulary and Ideas

Check ALL correct answers. Explain your choice and provide details.

1. Constructing a *perpendicular bisector* to segment can be:
   - I. Understood by reading Euclid's Elements.
   - II. Performed by using a ruler and compasses.
   - III. Accomplished by drawing a rhombus whose diagonal is the given segment.
   - IV. Achieved by drawing two isosceles triangles based on the given segment.
   - V. Done by finding the midpoint of the segment and erecting a perpendicular to the segment.
   - VI. All of the above.
2. To *bisect* a segment means:
   - I. To chop up the segment in half.
   - II. To be perpendicular to the segment.
   - III. To cross the segment in its midpoint.
   - IV. To draw another segment equal in length to the given segment.
   - V. More than one of the above.
3. The sentence "Two planets are *equidistant* from the sun.":
   - I. Refers to the elliptical orbits of some planets about the sun.
   - II. Is true in our solar system.
   - III. Should be corrected (how?).
   - IV. Makes sense.
   - V. The two planets and the sun are all at equal distances from each other.
   - VI. 40% of the above.

RECAP 2: Applications

- In what situations in real life might *perpendicular bisectors* help us? List at least three such situations.
- Find objects around the house that are *equidistant* from other objects.

RECAP 3: Geometric Visualization

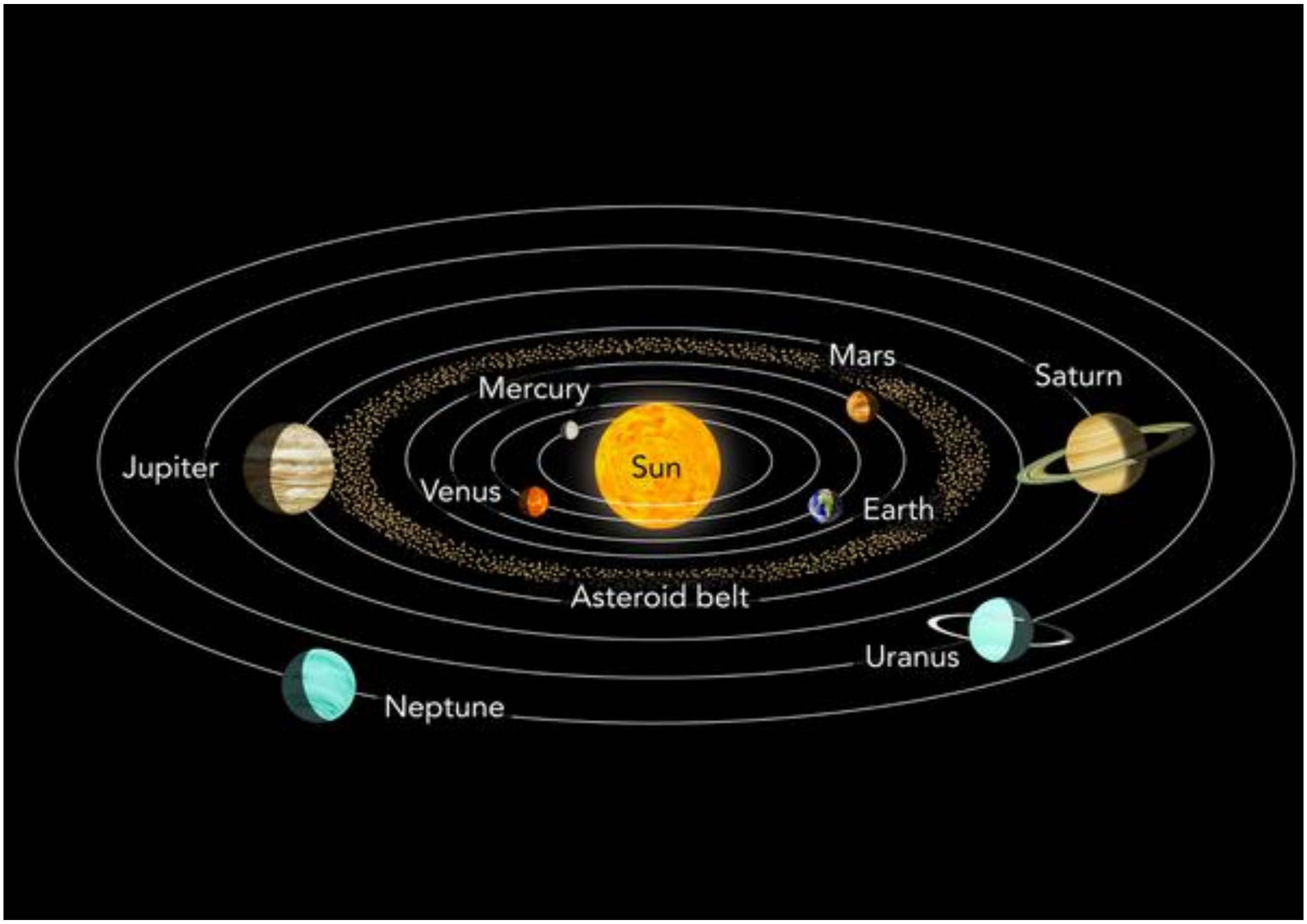
- Could *three* objects be all equidistant from each other? What shape would they form?
- How about *four* objects?
- How about *five* objects?

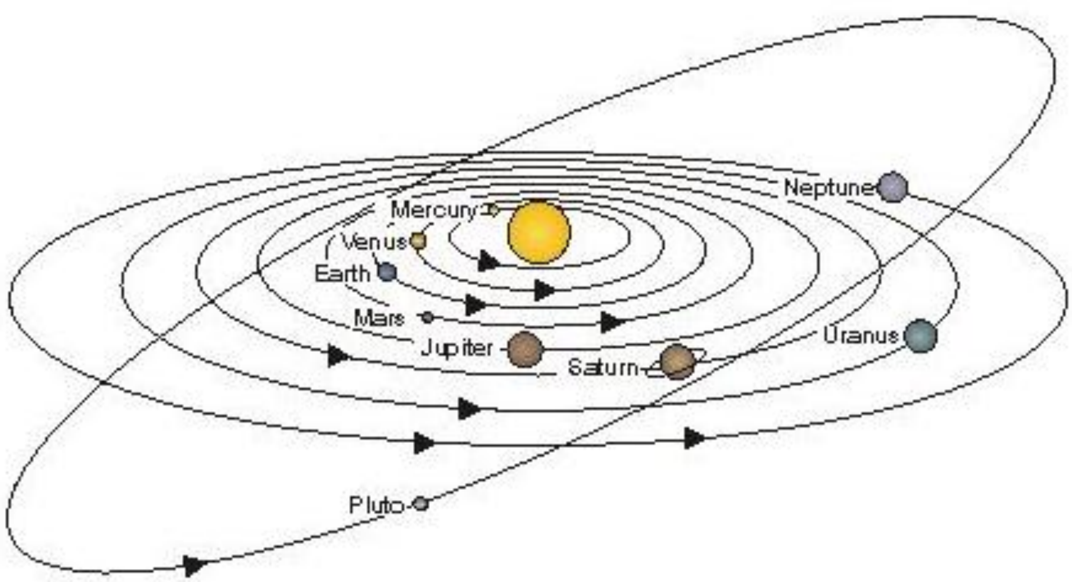
In each case, what space do you need to place the objects in?

RECAP 4: Origins of the Words Per-pendicular, Equi-distant, and Bi-sector

Connect each word or part of a word on the left with its meaning on the right, and color the two blocks the same way.

perpendicularis	to weigh, to hang
distantem	equal
bi	to cut
aequi	thoroughly
equidistant	to balance carefully
per	to cut in half
pendere	at the same distance
bisect	vertical, as a plumb line
perpendere	distant
secare	two


Mars
Saturn
Mercury
Jupiter
Sun
Venus
Earth
Asteroid belt
Uranus
Neptune


Neptune
Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Pluto