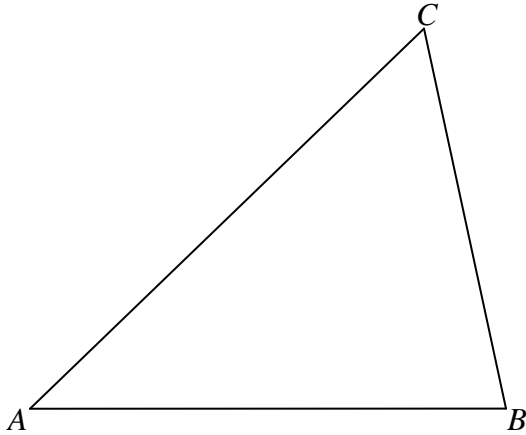# 5. Midsegments and Midtriangle
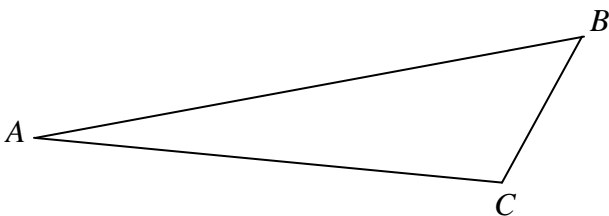
Problem 1. Find the *midpoints* of sides AB, BC, and CA and label them C_1, A_1, and B_1. Connect these three midpoints to form a new smaller triangle $A_1B_1C_1$.

Problem 3. Give a definition of:
   a) a parallelogram;
   b) a trapezoid.
Make sure that your definitions are as concise as possible: give just enough information to force someone to draw the quadrilateral of the required type.

A *parallelogram* is:

_____

_____

_____

A *trapezoid* is:

_____

_____

Definition 1. The segments A_1B_1, B_1C_1 and C_1A_1 are called the *midsegments* of triangle ABC. Triangle $A_1B_1C_1$ is called the *midtriangle* of the original triangle ABC.

Question 1. Which segments do you think are equal in your picture? Mark the equal segments with the same tick-marks.

Question 2. Color in 4 different colors the resulting 4 small triangles inside ABC. How many figures of the following type do you see in your picture from Problem 1:
   a) How many triangles? _____
   b) How many parallelograms? _____
   c) How many trapezoids? _____
List the figures below by their vertices.

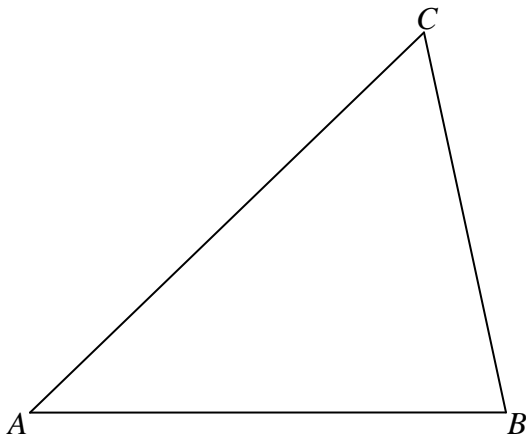**Problem 2.** Find the midpoints and test your conjecture about equal segments on the triangle below. Does it still hold?

Triangles: _____

_____

Parallelograms: _____

_____

Trapezoids: _____

_____

Definition 2. A *parallelogram* is a quadrilateral with two pairs of parallel opposite sides.

Definition 3. A *trapezoid* is a quadrilateral with (at least) one pair of parallel sides.

Problem 4. Once again, draw the midpoints of sides AC and BC and mark them with B_1 and A_1. Summarizing all your findings above, state what you believe is true regarding the midsegment B_1A_1 and the opposite side AB.

Midsegment B_1A_1 is always:

  a) _____ of side AB;

  b) _____ to side AB.

Confirm your conjecture for the other two midsegments and their opposite sides in $\triangle ABC$.

Question 3. Assuming your conjectures are correct, what can you say about the sides of the four small triangles in $\triangle ABC$? How about the midtriangle and the original big triangle ABC?

Theorem 1. A midsegment is parallel to and half of the length of the opposite side.

Corollary 1. The midtriangle has sides that are parallel to and half of the lengths of the corresponding sides of the original triangle.

Definition 4. *Congruent* triangles have the same shape: their corresponding sides are equal in length and their corresponding angles are the same too.

Corollary 2. The midsegments divide a triangle into:
  a) 4 *congruent* smaller triangles;
  b) 3 (overlapping) parallelograms;
  c) 3 (overlapping) trapezoids.
The parallelograms are not necessarily congruent, and neither are the trapezoids (why not?), but the triangles are congruent.

Question 4. How can you transform one of the four congruent small triangles to any other of these triangles? What operations, do you think, will preserve the shapes of figures?

Theorem 2. Congruent figures can be transformed into one another by the following operations:
  • *translation* (sliding);
  • *rotation* (revolving about a point);
  • *reflection* (flipping across a line).

Problem 5. Check that you can transform each of the four small congruent triangles into any of the others.

RECAP 1: New Vocabulary

What mathematical words (concepts) do you know that start with "mid" or "med"? List them and explain what they mean.

RECAP 2: Mathematical Logic

- What is the difference and what is the similarity between a *theorem* and a *corollary?* Which needs to be proven?

RECAP 3: True or False?

- A rhombus is a parallelogram: ___
- A parallelogram is a trapezoid: ___
- Trapezoids can't be parallelograms: ___
- A rhombus is never a parallelogram: ___
- Congruent triangles have equal corresponding sides: ___
- Rotating a triangle 180° about one of its vertices changes its shape: ___
- The midtriangle has the same shape and size as the other three triangles obtained by cutting out the midtriangle: ___
- Midtriangles have the same shape but not the same size as the original triangles: __
- A midsegment is always half the length of a nearby side of the triangle: ___

RECAP 4: Theory

What facts did we learn in this unit about:

- Midsegments in a triangle?
- The midtriangle of a triangle?
- Parallelograms?
- Trapezoids?
- Transformations that do not change the shape of a figure?
- Can you think of transformations that change the size but not the shape of a figure? How about changing the shape but not the size of a figure?

RECAP 5: Think/Experiment Ahead

- What will happen if you fold the big triangle inwards along its three midsegments? Will each of the outside small triangles fold exactly onto the midtriangle? Why or why not?
- Cut out a paper model of the triangle and perform the folding to verify if your guesses are right.

RECAP 6: New/Old Vocabulary

Think of at least 5 math and science words (concepts) you know that end with "oid". List them and explain what they mean.