

# 5. Midsegments and Midtriangle: Answer Key

## RECAP 1: New Vocabulary (25 pts)

What mathematical words (concepts) do you know that start with “mid” or “med”? List them and explain what they mean.

**5 pts per item:** 2 pts for word, 3 pts for explanation

1. *Midpoint*: the point in the middle of a segment (or a side of a triangle); the point halfway between two other points.
2. *Median*: the segment connecting a vertex with the midpoint of the opposite side of a triangle.
3. *Medicenter*: the intersection of the three medians in a triangle.
4. *Midsegment*: the segment connecting the midpoints of the two sides of a triangle.
5. *Midtriangle*: the triangle whose vertices are the midpoints of the sides of the original triangle; or the triangle whose sides are the midsegments of the original triangle.

## RECAP 2: Mathematical Logic (10 pts)

- What is the difference and what is the similarity between a *theorem* and a *corollary*? (8 pts)
  - A theorem and corollary are both true statements. (A corollary is a theorem, only called differently because of us, human beings, who like to distinguish between things by more features.)
  - A corollary is a consequence of another true statement (be it another theorem, corollary, or other).
- Which needs to be proven? (2 pts)
  - *Both* need to be proven.

## RECAP 3: True or False? (18 pts)

**2 pts per item.**

- A rhombus is a parallelogram: T
- A parallelogram is a trapezoid: T
- Trapezoids can't be parallelograms: F
- A rhombus is never a parallelogram: F
- Congruent triangles have equal corresponding sides: T
- Rotating a triangle  $180^\circ$  about one of its vertices changes its shape: F
- The midtriangle has the same shape and size as the other three triangles obtained by cutting out the midtriangle: T
- Midtriangles have the same shape but not the same size as the original triangles: T
- A midsegment is always half the length of a nearby side of the triangle: F

## RECAP 4: Theory (18 pts + 2 pt bonuses)

What facts did we learn in this unit about:

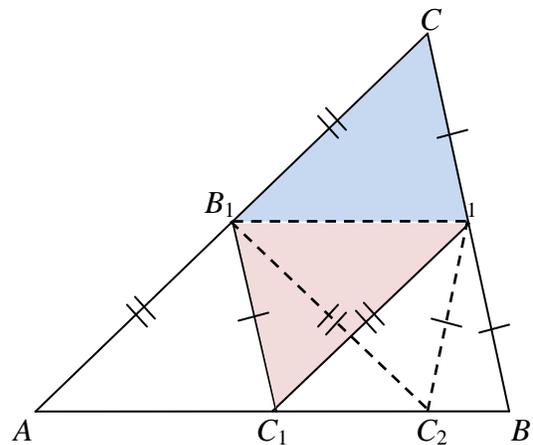
- *Midsegments* in a triangle? (3 pts)
  - Are parallel to and half the size of the opposite sides of the triangle.
  - Divide the triangle into four congruent triangles, three overlapping parallelograms, and three overlapping trapezoids.
- The *midtriangle* of a triangle? (3 pts)
  - Is congruent to the other three triangles which, together with the

- midtriangle, make up the original triangle.
- Has sides that are parallel to and half the size of the sides of the original triangle.
- Parallelograms? (3 pts)
  - A parallelogram is a quadrilateral with two pairs of parallel sides.
- Trapezoids? (3 pts)
  - A trapezoid is a quadrilateral with two (or more) parallel sides.
- Transformations that do not change the shape of a figure? (3 pts)
  - Translations, reflections, and rotations do not change the shape of a figure.
- Can you think of transformations that change the size but not the shape of a figure? How about changing the shape but not the size of a figure? (3 pts)
  - Rescaling (zooming in or out) does not change the shape of a figure but changes its size.
  - Lots of transformations would work here. For example, you cut the figure in two and glue the two parts differently: you will likely change the shape of the figure, but its total area will remain the same.

**RECAP 5: Think/Experiment Ahead**  
(9 pts)

- What will happen if you fold the big triangle inwards along its three midsegments? Will each of the outside small triangles fold exactly onto the midtriangle? Why or why not? (6 pts)

- The outside triangles will not necessarily fold onto the midtriangle. For example, on the picture below, the blue “outside” triangle will not fold onto the pink midtriangle. Instead the blue triangle will fold onto the *dashed* triangle.
- The reason for this is that the blue and the pink triangles form a parallelogram, while the blue and dashed triangles form a deltoid (a kite), and the two quadrilaterals are not the same!
- The only situation when the blue triangle will fold onto the pink one is when  $CA_1 = A_1C_1$ , i.e., the blue-pink parallelogram is a rhombus, i.e.,  $CA_1 = CB_1$ , i.e.,  $CB = CA$ , i.e., the original triangle is isosceles. For all outside triangles to fold onto the midtriangle, we have an equilateral original triangle  $ABC$ .



- Cut out a paper model of the triangle and perform the folding to verify if your guesses are right. (3 pts)

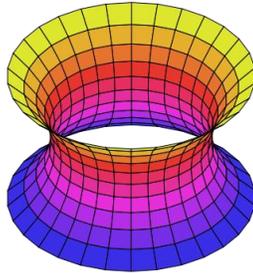
**RECAP 6: New/Old Vocabulary  
(20 pts + 2 pts bonuses)**

Think of at least 5 math and science words (concepts) you know that end with “oid”. List them and explain what they mean.

**4 pts per item:** 2 pts for word, 2 pts for explanation

**Math words:**

1. *Catenoid*: a surface in space arising by rotating a *catenary* curve about its axis; found and studied by Leonhard Euler in 1744.



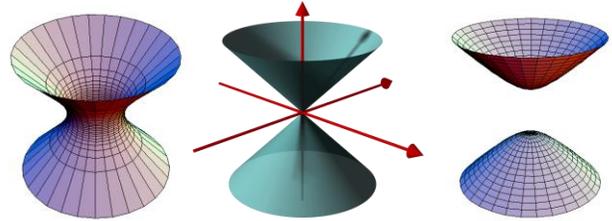
A hanging chain (or a spider web) forms a *catenary* curve.

2. *Centroid*: the intersection of the three medians in a triangle.
3. *Deltoid*: a quadrilateral with two pairs of equal adjacent sides. It looks like a *kite*.
4. *Ellipsoid*: a surface in space which is the analogue of the ellipse in the plane. An U.S. football has the shape approximately of an ellipsoid.

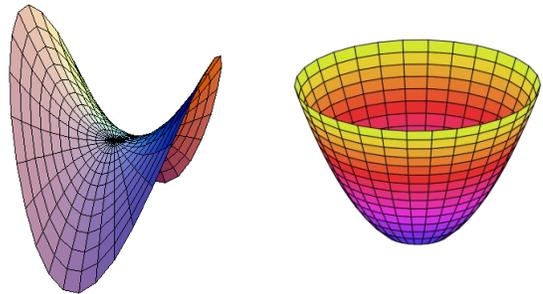


Infosys Ellipsoid (Pune, India)

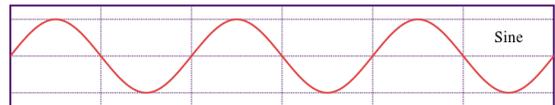
5. *Hyperboloid*: a surface in space which can take the shape of double-sided bottomless vase, a double-sided cone, or two symmetric infinite cups.



6. *Paraboloid*: a surface in space which can take the shape of an infinite saddle or cup.



7. *Sinusoid*: a curve that describes the smooth repetitive oscillation of the sine function.



8. *Trapezoid*: a quadrilateral with two parallel sides.

**Science words:**

9. *Alkaloid*: naturally occurring chemical compounds that contain mostly basic nitrogen atoms; produced by bacteria, fungi, plants, and animals.

The first individual alkaloid, morphine, was isolated in 1804 from poppy.



10. *Android*: robot designed to look like a human.



**Everyday words:**

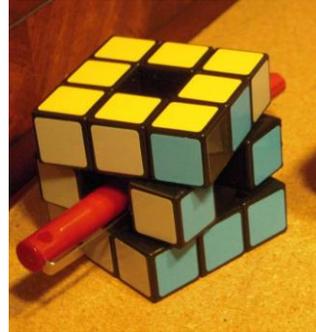
13. *Avoid*: keep away from something/someone or stop oneself from doing (something)

14. *Paranoid*: having or showing an unreasonable feeling that people are trying to harm you or do not like you.

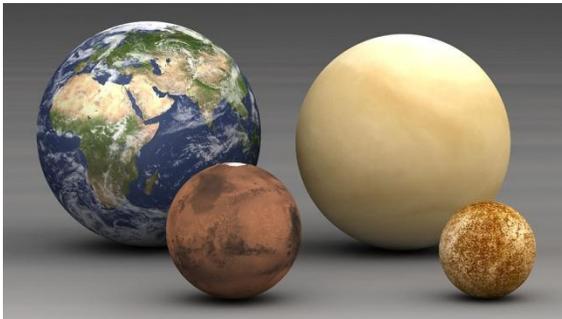
15. *Void*: not valid, completely empty.

11. *Asteroid*: minor planet in the *inner Solar System*.

The inner Solar System comprises the asteroids and the *terrestrial* planets: Earth, Mars, Venus, and Mercury:



*Void* Rubik's 4 x 4 x 4 and 3 x 3 x 3 cubes



(shown above from left to right, sizes to scale, inter-planetary distances not).

12. *Droid*: robotic machines from Star Wars, books and television series.

