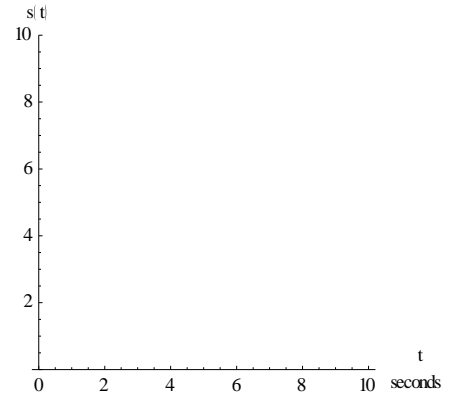


Feel the heat – the trek to Partial Differential Equations
Warm-up problems
Berkeley Math Circle, 2014-01-21
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For each of the following four problems, even if you are not able to prove an exact solution, try to graph and/or explain how the solutions should behave (and why.)

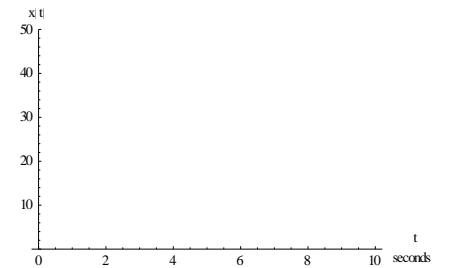
1) (Very easy) “calculus”:

You are stopped at a red light while on your bicycle. For the first 10 seconds after the light turns green, you accelerate (go faster) at a constant rate so after each second you end up going 1 meter per second faster. Fill in the graph (and equation) for your speed $s(t)$ as a function of time.



2) **Actual calculus:**

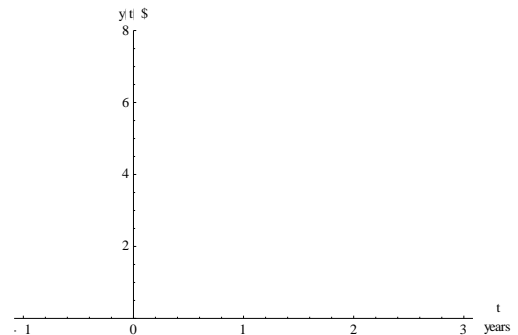
- a) Now that you know from problem 1 what your speed is as a function of time, figure out the total distance $x(t)$ you have gone – again as a function of time for t between 0 and 10 seconds.
- b) If you had started with the answer to question 2, how would you get the answer to question 1?
- c) What are the units of the y-axis in questions 1 and 2?



3) **Ordinary differential equations** (“ODE”, where “ordinary” means single-variable):

You got a wonderful investment tip that lets you invest your money $y(t)$ so that the rate at which money is added to your account (“interest”) equals one times $y(t)$. This increases $y(t)$, which in turn increases how quickly money is added to your account. After your initial investment $y(0)$, the only thing that changes $y(t)$ is interest.

- a) Can you find a way to describe this situation in math notation? If so, you have managed to state the “equation” or “problem”.
- b) Any function $y(t)$ that satisfies the conditions of the problem is called a “solution.” Graph a solution or two (if you can think of more than one.)
- c) Do you have enough information to allow you to determine $y(t)$? If not, what's missing?



4) **Partial differential equations** (“PDE”, where “partial” means you have “multivariate” functions, i.e. depending on more than one variable.)

Bob holds on to the left end of 10-cm steel rod whose other end is being heated by a flame.

- a) Discuss with your personal injury lawyer some of the relevant theories of liability (just kidding.)
- a) Really: your goal is to understand the temperature $h(x, t)$ of the rod – both as a function of position x (from 0 to 10 cm) and of time t (starting at time $t=0$ seconds.) Say the rod starts out at 0 degrees Celsius at time $t=0$, except that the right end of the rod is being heated by a flame burning at 1000 degrees. Draw graphs of how the rod's temperature should look say at times $t=0$, $t=1$, and $t=10$.

