

RECURSION

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Recursive Relationship

Problem 1 Given a staircase of n steps.

- each time you take 1 step or 2 steps. How many different ways to get to the top? $a_n = a_{n-1} + a_{n-2}$
- each time you take 1, 2, 3 steps. How many different ways to get to the top? $a_n = a_{n-1} + a_{n-2} + a_{n-3}$.

Problem 2 Given n countries aligned on a circle. We have three colors. Any neighboring countries can not have the same color. What is the total number of different ways to color them? ($a_n = 3 \cdot 2^{n-1} - a_{n-1}$)

Problem 4 Given n squares aligned on a line. We have two colors (red and blue) to color them. We can not have two red squares next to each other. How many different ways to color the n square. ($a_n = a_{n-1} + a_{n-2}$)

Problem 5 Given a n by n grids, we can go up or right at each step. How many different route can we go from $(0, 0)$ to (n, n) ? ($a_{i,j} = a_{i-1,j} + a_{j,i-1}$)

Solving Recursive Relationship

Problem 4 Suppose $a_n = 2a_{n-1}$ and $a_1 = 1$.

Problem 5 Suppose $a_n = pa_{n-1}$ and $a_1 = 1$,

Problem 6 Suppose $a_n = a_{n-1} + 1$ and $a_1 = 1$

Problem 7 Suppose $a_n = a_{n-1} + p$ and $a_1 = 0$

Problem 8 Suppose $a_n = 2a_{n-1} - 1$ and $a_0 = 1$

Problem 9 Suppose $a_n = 2a_{n-1} + 1$ and $a_0 = 1$

Problem 10 Suppose $a_n = 3a_n + 1$ and $a_0 = 1$.

Problem 11 Suppose we have that $a_{n+1} = 2a_n - a_{n-1}$ and $a_1 = 1$ and $a_2 = 2$.