## BULGARIAN MATH OLYMPIADS

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## Berkeley Math Circle – Beginners Group September 3, 2013

## Tasks throughout this session. For each problem:

- (A) Solve the problem. If possible, solve it in more than one way.
- (B) What area(s) of mathematics does the problem belong to?
- (C) What math objects and concepts are involved in the problem and its solution?
- (D) What problem-solving techniques (PSTs), ideas, formulas, properties, theorems, or axioms did you use to solve the problem?
- (E) Can you make the problem harder? Can you generalize it? Can you create a new, interesting problem related to the given one?

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(1) What is the value of the expression  $\left(3\frac{3}{5} \cdot 5\frac{7}{9} - 3\frac{5}{9} \cdot 3\frac{3}{5}\right) : 1\frac{1}{4}$ ?

(2) One fourth of which number equals 4% of 50?

- (3) What is the unknown number x from the equation  $(5-4x) \cdot 1\frac{2}{7} = 3\frac{2}{5} + 2.6?$
- (4) A  $4 \times 4$  square table is filled with the numbers 1, 2, 3, and 4 in such a way that every number appears once in every row, column, and diagonal. What number is A?

1	2	3	4
A			
	1	4	3

- (5) 5 teams participated in a basketball tournament. Each team played every other team exactly once. How many games were played in total?
- (6) The 5-digit number  $\overline{21 * 36}$  is divisible by 3. What is the product of all possible values of \*?
- (7) Two identical baskets full of apples weight the same. If we move 8.2 kg apples from the first to the second basket, then the apples in the second basket will double. How many kilograms apples are there in each basket?

- (8) A baker sold  $\frac{3}{4}$  of all croissants in the morning and  $\frac{2}{3}$  of the rest in the afternoon, and in the evening only 8 were left. How many croissants did he sell overall?
- (9) What is the sum  $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6}$  equal to?
- (10) Several books must be packed. If in each packet we put 3, or 4, or 5, or 6 books, there will always be one unpacked book; but if in each packet we put 7 books, then all books will be packed. What is the smallest possible number of books that we need to pack?
- (11) To feed a total of 56 dogs and cats we need 304 sausages. Every cat eats 5 sausages and every dog eats 6. How many are the cats?
- (12) In  $\triangle ABC$ , M is the midpoint of BC, L is the midpoint of AM, and K is the midpoint of AB. If the area of  $\triangle ABC$  is 36 sq. cm, then how much is the area of  $\triangle KBL$ ?
- (13) Consider all prime numbers that are not larger than the largest common factor of 150 and 180. How many of these numbers are written in digits which are themselves prime numbers?
- (14) When baked, cashews lose 25% of their weight. How many kilograms of raw cashews must be baked to have 12 kg of baked cashews?
- (15) A handyman can complete an order in 5 hours, and his apprentice in 2 more hours. The two worked together 1 hour. What part of the order remained unfinished?
- (16) The product of 100 natural numbers equals 100. Find all possible sums of these 100 numbers.

MATHEMATICS COMPETITION FOR \_\_\_\_\_GRADE, APRIL 29 2006



- (17) How many triangles are there in the first drawing in the picture?
- (18) Which number will you write in the place of X in the second drawing? Why?
- (19) If you find the rule which the first two circles oblige in the third drawing, you will quickly find the missing number in the third circle. What number is this?

- (20) If  $A = 1 + 3 + 5 + \dots + 2001 + 2003$  and  $B = 2004 + 2002 + \dots + 6 + 4 + 2$ , how much is the difference B A?
- (21) How many are all 4-digit numbers the sum of whose digits is 4?
- (22) The difference of two numbers is 29. How will it change if we subtract 15 from the minuend and we add 14 to the subtrahend? (Minuend subtrahend = difference.)
- (23) A boy and a pig weigh as much as 5 sheep. Two cats and a pig weigh as much as 3 sheep, and one pig weighs as much as 4 cats. As much as how many cats does one boy weigh?
- (24) A steamer traveled 7 hours, 3 of which along a lake with speed 27 km/h, and then along a river which goes into the lake. Find the distance traveled by the steamer for these 7 hours if the speed of the river is 3km/h.
- (25) In a rectangular garden with length 40m and width 25m there are four alleys with width 5m each, as shown on the diagram. What is the area of the alleys?



- (26) In an office there are 14 desks with one, two, or three drawers. The total number of drawers is 25. The desks with one drawer are as many as the tables with two and three drawers altogether. How many tables with three drawers are there?
- (27) If same digits correspond to same letters, replace the letters with digits so that the equation  $\overline{xyzt} + \overline{xyz} + \overline{xy} + x = 2004$  is satisfied.



(28) Put the seven pieces above next to each other in the seven places to form an equation.