

Berkeley Math Circle
HOW TO GAMBLE IF YOU MUST

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- 1 It costs a consumer \$1 to buy a Klopstockia lottery ticket. The buyer then scratches the ticket to see the prize. Compute, to the nearest penny, the expected profit that the state of Klopstockia makes per ticket sold, given the following scenarios for prizes awarded. (The state will make a profit if the expected value of the lottery ticket is *less* than \$1.)

(a)

Prize	\$1	\$20
Probability	$\frac{1}{10}$	$\frac{1}{1,000}$

(b)

Prize	\$1	\$20	a free lottery ticket
Probability	$\frac{1}{10}$	$\frac{1}{1,000}$	$\frac{1}{5}$

- 2 Two players alternately toss a penny, and the one that first tosses heads wins. What is the probability that

- (a) the game never ends?
- (b) the first player wins?
- (c) the second player wins?

- 3 *What a Loser!* You arrive in Las Vegas with \$100 and decide to play roulette, making the same bet each time, until you are either bankrupt or have doubled your money. Which of the following strategies is best?

- (a) Making bets of \$1 each time.
- (b) Making bets of \$10 each time.
- (c) Making a single bet of \$100.

- 4 *A Gambling "System."* Suppose you are playing a game with a 50% chance of winning each time. You can bet any amount, and if you win, you win twice your bet. If you lose, you lose your bet. In other words, if you bet B dollars, your *profit* is $\pm B$ depending on whether you win or lose. You decide that you will play, stopping as soon as you win, doubling the size of your bet each time. Are you guaranteed to make a profit? What if you triple instead of double?

- 5 *The St. Petersburg Paradox.* Consider the following game. I will flip a fair coin until it shows up heads. We keep track of the number of flips until this happens. If it happens on the first flip, I'll pay you \$2. If it takes two flips, then I'll pay you \$4. Three flips, \$8, etc. In other words, if it takes n flips until the first head, I will pay you 2^n dollars. Pretty sweet game!

How much is this game worth *to you*? In other words, if there was a ticket that allowed you to play the game once with me (I flip the coin until it is heads, and pay you the appropriate amount), how much would you pay for the ticket? Clearly, you'd pay at least 1 dollar. In fact, you'd almost certainly pay 2 dollars. How about 3? 4? 5? More?