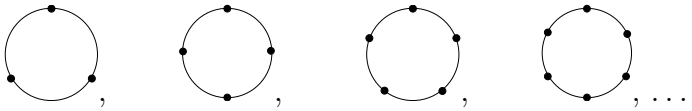
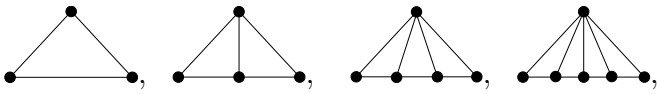
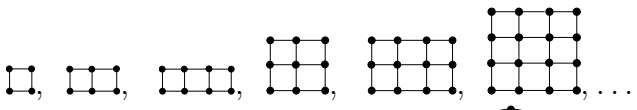
THE GOOD WILL HUNTING PROBLEM AND RELATED COMBINATORICS

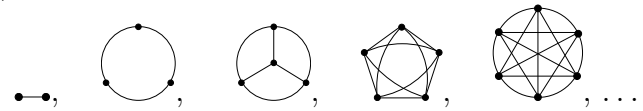
BERKELEY MATH CIRCLE - SEPTEMBER 18, 2012 (GREGG MUSIKER)

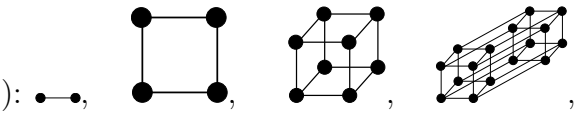
Families of Graphs:

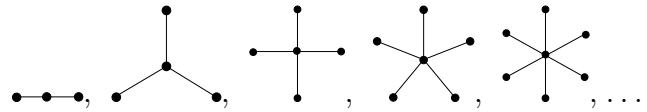
Cycle Graphs (C_n for $n \geq 3$): 

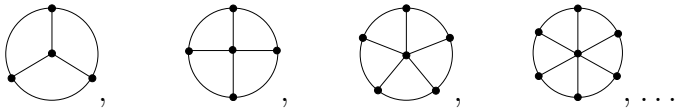
Fan Graphs (F_n for $n \geq 2$): 

Grid Graphs ($G_{n,m}$ for $m \geq n \geq 2$): 

Complete Graphs (K_n for $n \geq 2$): 

Hypercube Graphs (Q_n for $n \geq 1$): 

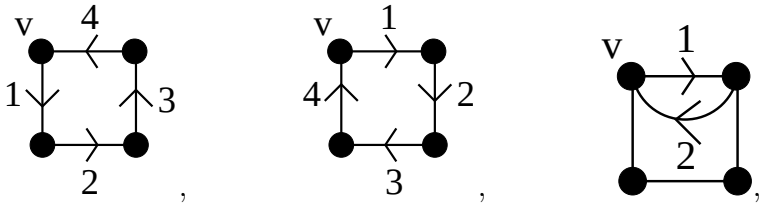
Star Graphs (S_n for $n \geq 1$): 

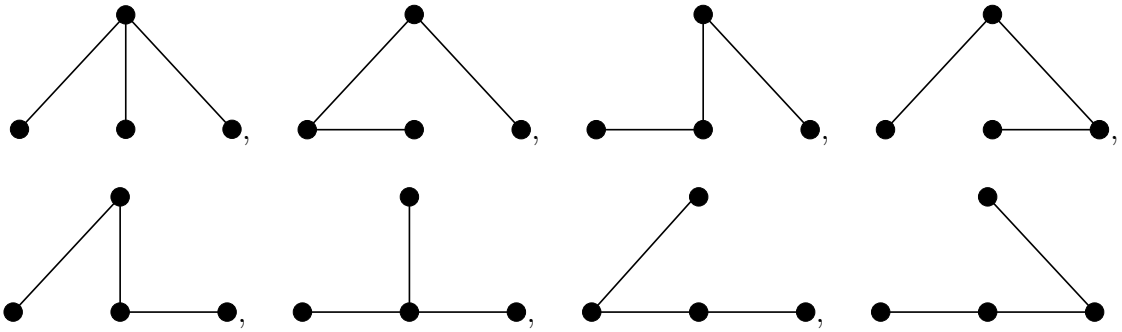
Wheel Graphs (W_n for $n \geq 3$): 

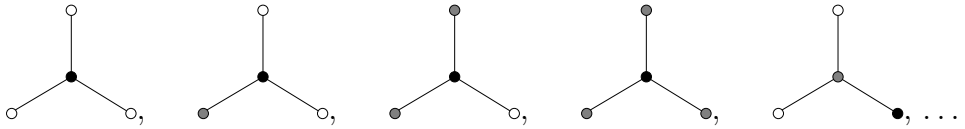
A Selection of Enumerative Questions: What are the number of . . . ?

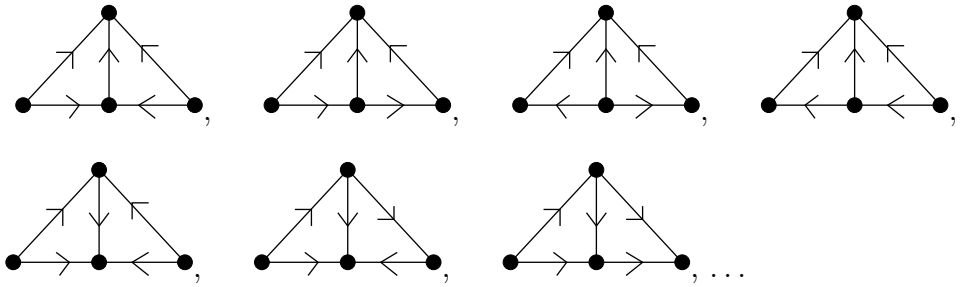
(1) Closed Walks
(2) Spanning Trees
(3) Proper Colorings
(4) Acyclic Orientations
(5) Perfect Matchings

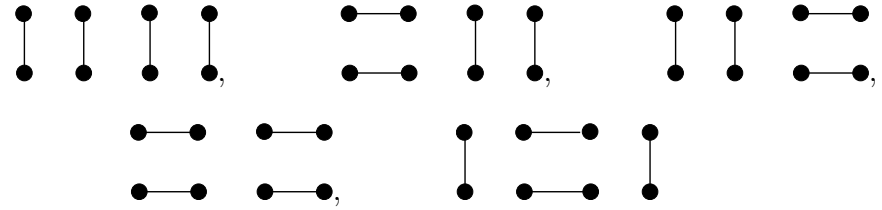
We can use algebraic techniques to solve these questions!

(1) 

(2) 

(3) 

(4) 

(5) 

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Lecture Summary: Three questions from algebraic combinatorics

(1) How many spanning trees are in the nth hypercube, Q_n?

Answer:

$$\frac{1}{2^n}\prod_{i=1}^{n}(2i)^{\binom{n}{i}}.$$

This led to a discussion of counting walks in graphs using *Adjacency Matrices* and counting spanning trees by using the *Matrix-Tree Theorem*. More details are available at `http://www.math.umn.edu/~musiker/4707/Matrices.pdf`, and Chapters 1, 9 of `http://math.mit.edu/~rstan/algcomb.pdf`.

(2) What is the relationship between *Chromatic polynomials* and *Acyclic Orientations (Tournaments)*?

What are the number of proper colorings (with x colors) or acyclic orientations of the following families of graphs:

complete graphs K_n, cycle graphs C_n, and n-vertex trees?

(3) The generous or maniacal teacher?

If all 50 students find their names after opening 25 boxes, they get no homework. Otherwise, everyone gets a pop quiz. What is there chance of having no homework if they follow the optimal strategy?

See Section 12.1 of `http://math.mit.edu/~rstan/algcomb.pdf` or Problem 1 of `http://www.math.dartmouth.edu/~pw/solutions.pdf`.

Some Other Recommended References:

`http://ocw.mit.edu/courses/mathematics/18-312-algebraic-combinatorics-spring-2009/`.

A Walk Through Combinatorics: An Introduction to Enumeration and Graph Theory by M. Bona

Combinatorics and Graph Theory (Undergraduate Texts in Mathematics) by J. Harris, J. Hirst, and M. Mossinghoff

Of a different flavor but also contains algebraic combinatorics:

Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks by P. Diaconis and R. Graham