# Berkeley Math Circle
# Monthly Contest 8
# Due May ??, 2012

**Instructions**

This contest consists of 7 problems, some of which are easier than the others. Problems 1–4 comprise the Beginner Contest (for grades 8 and below) and Problems 3–7 comprise the Advanced Contest (for grades 9–12). Every problem is worth 7 points. Please write your solution to every problem on a separate sheet of paper, and on top of each sheet include your name, grade, school, and BMC level, as well as the problem number and the contest number. Thus, the header on each sheet should look something like:

Solution to Problem 3 of BMC Monthly Contest 5
by Bart Simpson
in grade 7, BMC Intermediate
from Springfield Middle School, Springfield

If you submit more than one sheet for a specific problem, please, staple the sheets together to avoid getting them confused with someone else's solution. Please, do NOT staple together solutions to DIFFERENT problems, as they will be graded separately.

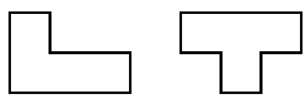
Carefully justify your answers to avoid losing points. Include all relevant explanations in words and all intermediate calculations. Answers without justification will receive no credit. However, good reasoning with minor calculational errors may receive a lot of points. Thus, submit solutions to as many problems as you can since partial credits will be awarded for sufficient progress on any particular problem.

Remember that you are NOT ALLOWED to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, or other means of communication. You can consult any book that you wish. For more on the contest rules, please, check the BMC website at http://mathcircle.berkeley.edu.

Enjoy solving these problems and good luck!

**Problems**

1. Determine whether it is possible to tile a standard $8 \times 8$ chessboard with 15 L-tiles and 1 T-tile of the shapes below:



(Each tile covers four squares of the chessboard. The tiles can be flipped and rotated at will.)

2. A point $P$ lies inside a regular hexagon $ABCDEF$. The distances from $P$ to the sides $AB$, $BC$, $CD$, $DE$, $EF$, and $FA$ are respectively 1, 2, 5, 7, 6, and $x$. Find $x$.

3. Let $k$ be a positive integer. Prove that there is a positive integer $N$ with the following properties:

   (a) $N$ has $k$ digits, none of which is 0.

   (b) No matter how the digits of $N$ are rearranged, the resulting number is not divisible by 13.

4. Suppose that $b$ and $c$ are real numbers such that the equation

$$x^2 + bx + c = 0$$

   has two different solutions $x_1$, $x_2$. Suppose that

   (a) The (positive) difference between $x_1$ and $x_2$ is 1;

   (b) The (positive) difference between $b$ and $c$ is also 1.

   Find all possible values of $b$ and $c$.

5. Prove that any prime which is the difference of two cubes is also the sum of a square and three times a square.

   *Remark.* By *cube* and *square* are meant, respectively, the cube and square of a natural number.

6. Given five nonnegative real numbers with sum 1, prove that it is possible to arrange them at the vertices of a regular pentagon such that no two numbers connected by a side of the pentagon have product exceeding $1/9$.

7. In triangle $ABC$, $\angle A = 60^\circ$. Let $E$ and $F$ be points on the extensions of $AB$ and $AC$ such that $BE = CF = BC$. The circumcircle of $ACE$ intersects $EF$ in $K$ (different from $E$). Prove that $K$ lies on the bisector of $\angle BAC$.