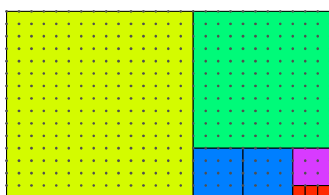


Continued Fractions from a Geometric Viewpoint

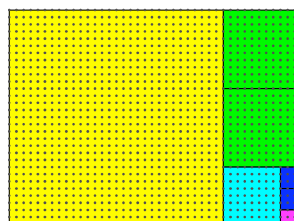
Ted Courant
 Berkeley Math Circle
 January 18, 2011
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- The dimensions of a football field in square feet: 120 yards \times 160 feet = 57600 square feet. One acre is 43,500 square feet.
 Reduce $\frac{43500}{57600}$ to get $\frac{121}{160}$, which is approximately $\frac{3}{4}$. So a football field is $\frac{4}{3}$ of an acre, or so.
 Find the partial quotients of $\frac{121}{160}$.
 Find a “best” approximation to an acre as a fraction of a football field.
- Use the Euclidean algorithm to find $\gcd(41,29)$; repeat for $\gcd(41,30)$ and $\gcd(26,15)$.
- Find all solutions to $41x - 29y = 1$, $41x - 30y = 1$ and $26x - 15y = 1$ (Hint: see problem 2)
- What is the area of a triangle with sides 13, 14, 15?
 Can you find other triangles with consecutive sides and integer area?
- Are any triangular numbers exactly double other triangular numbers?

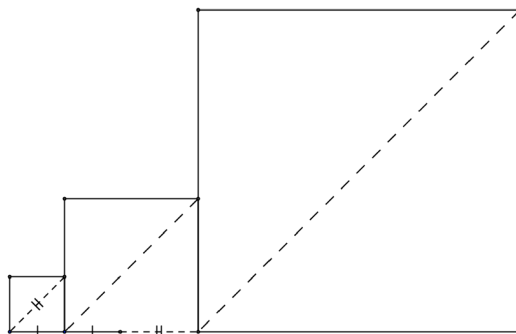
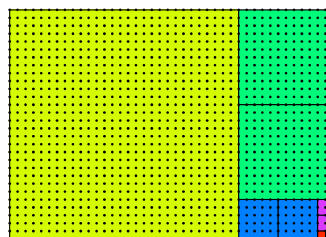
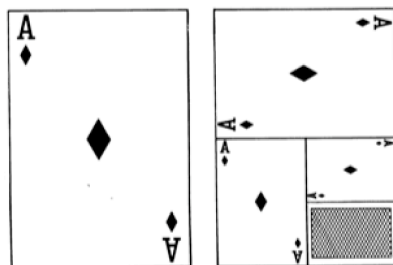
$$26 \times 15$$

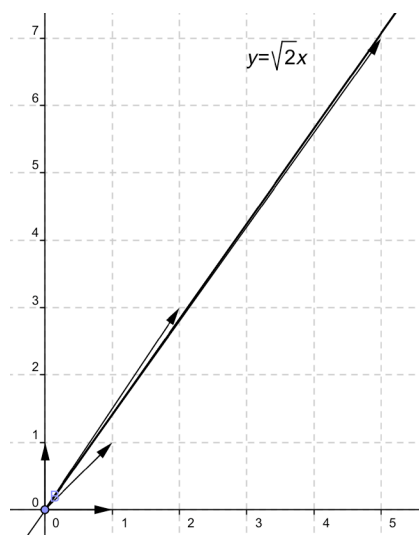
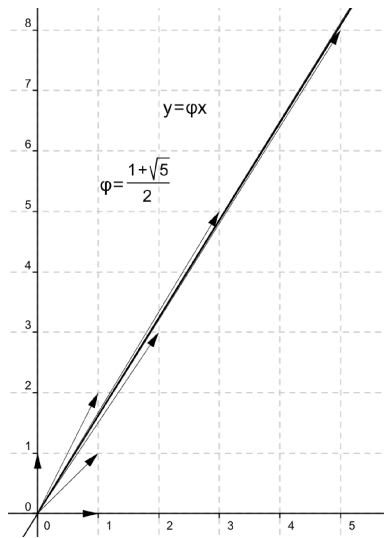
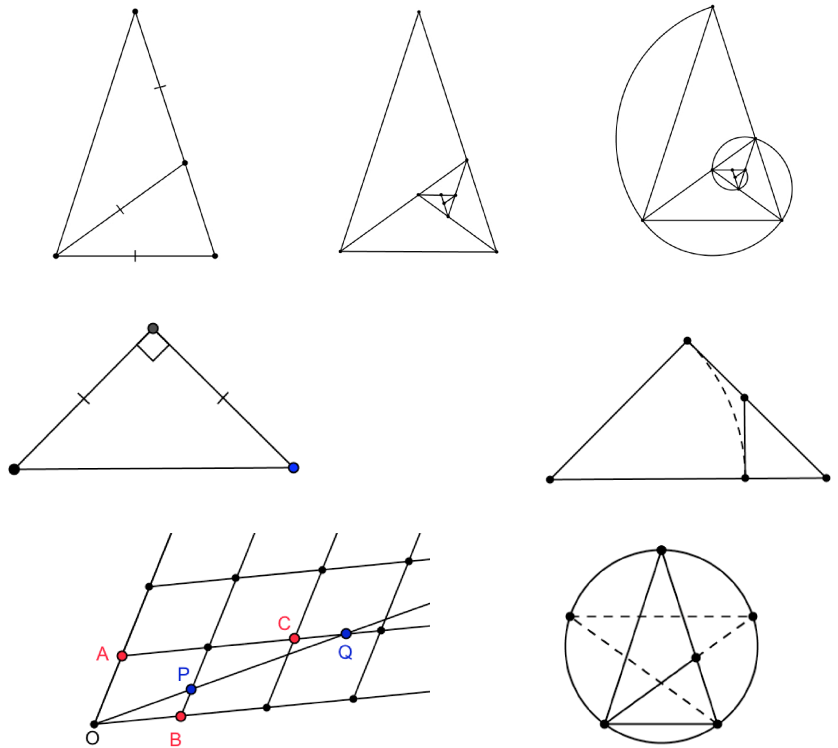


$$41 \times 30$$



$$41 \times 29$$





Sources

Excursions in Number Theory by C. Stanley Ogilvy (Dover Book republication (1988))

Real Numbers and Fascinating Fractions (based on N. M. Beskin's book: URL: kr.cs.ait.ac.th/~radok/math/mat4/start.htm)

Continued Fractions, by C. D. Olds (New Mathematics Library, MAA)

Solving the Pell Equation, by H. W. Lenstra Jr., Notices of the AMS, vol 49, number 2 (2002)

A Problem of Astronomical Proportion, by P. Harvey, The Mathematical Gazette vol 60, number 414, (1976)