Combinatorics - VII

Berkeley Math Circle - Beginner's

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Many of these problems are from *Mathematical Circles (Russian Experience)* and from *A Decade of the Berkeley Math Circle -Volume 1*

1. You toss a coin 5 times. How many different sequences of heads and tails can you get?

2. How many ways are there to sew a three-colored flag with three horizontal stripes of equal height if you have six colors of fabric? We can distinguish the top of the flag from the bottom.

3. My son Connor will be taking 6 classes next year: English, Drama, Teacher's Aid, Physics, Statistics, and Government. The schedule at his high school has 7 periods, so he will have one period free. How many different possible schedules could he have?

4. Eight points are marked on a plane so that no three of them are on the same straight line. How many triangles are there with vertices at these points?

5. In how many ways can you choose a debate team of 6 students from Ms. Jewls' class of 11 girls and 17 boys, if the team has to consist of 3 girls and 3 boys?

6. How many six digit numbers have at least one even digit? (Note: a number like 064862 with 6 digits that starts with the digit 0 is considered a 5 digit number, not a 6 digit number.)

7. There are three rooms in a dormitory: one single, one double, and one for four students. How many ways are there to house seven students in these three rooms?

8. Ten points are marked on a straight line, and 11 points are marked on another line, parallel to the first one. How many triangles can you make with vertices at these points?

9. You roll a 6-sided die 3 times. Among all possible outcomes, how many have at least one occurrence of the number 6?

10. How many different four-digit **even** numbers can be written using the digits 1, 2, 3, 4, if each digit can be used only once?

11. How many different four-digit **even** numbers can be written using the digits 1, 2, 3, 4 if each digit can be used any number of times?

12. You flip a coin 6 times. Of all the possible outcomes, how many have more heads than tails?

13. How many ways are there to distribute 3 one dollar bills and 10 quarters among 4 different children? Some children may get nothing. (Hint: remember the dogs and biscuits problem. You may want to distribute the dollars first, then the quarters.)

14. How many ways can you pick out an outfit to wear, if you have 5 shirts, 3 pairs of pants, and 12 pairs of shoes? (An outfit is one shirt, one pair of pants, and one pair of shoes.)

15. How many different types of milkshake can you make if a milkshake must contain exactly 3 different flavors of ice cream and you have 10 flavors total in your kitchen?

16. How many ways can you play 5 songs in a row on your iPod, if you have 15 songs total, and you don't want to repeat any songs?

17. Each of two novice collectors has 20 different stamps and 10 different postcards. We call an exchange fair if they exchange a stamp for a stamp or a postcard for a postcard. How many ways are there to carry out one fair exchange between these two collectors?

18. The rules of a soccer tournament say that each team must play each other team exactly once. How many games will be played if there are 18 teams?

19. How many "words" can be written using exactly 5 letters A and 4 letters B? (Here, a "word" is just any arrangement of letters, like ABBABAAAB, for example.)

20. There are 12 different dogs, 8 different cats, and 4 different parakeets at a pet store. How many ways are there to choose two pets of different species?

21. How many ways are there to pick 1 or more math books to take on vacation if you have a shelf of 10 books to choose from?

22. You are organizing a talent show with 8 different acts. In how many different orders can the acts be arranged, if the fire-eating unicycle rider has to be scheduled in the first half because she has to leave early?

23. How many ways are there to represent the number 10 as a sum of 5 non-negative integers? The order of the numbers matters here, so, for example, $1 + 4 + 4 + 0 + 1$ is considered different from $0 + 1 + 1 + 4 + 4$. Hint: remember the dogs and biscuits problem.

24. *How many ways are there to represent the number 10 as a sum of 5 positive integers? The order of the numbers matters here, so, for example, $1 + 4 + 3 + 1 + 1$ is considered different from $1 + 1 + 1 + 3 + 4$.

25. *How many ways are there to rearrange the letters in the word "FLAMINGO" so that the vowels will be in alphabetical order and so will the consonants? For example, FAGILMON (A - I - O, F - G - L - M - N).

26. *How many ways are there to write 10000 as a product of 3 numbers, if the order of the factors matters? For example, 500*4*5 is considered different from 4*5*500.