Numbers and Sequences part 2:
Some Of My Favorite Sequences

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Problem 1. In a list of integers, there is no set of consecutive terms such that the product of those terms is a perfect square. How long can the list be? What restrictions can we put on this problem to make it interesting?

Problem 2. Given an integer n, let $a(n)$ be the smallest number m such that there exists an increasing sequence of integers beginning with n and ending with m such that the product of the integers in the sequence is a perfect square. For instance, $a(2) = 6$ because 2, 3, 6 works and no smaller number than 6 can be the final number. What pattern do you notice about these $a(n)$? What properties do they have?

Problem 3. Does there exist a sequence such that the sequence, along with its first differences, contains every positive integer exactly once? If so, how many such sequences are there? What if the differences have to be increasing?

Problem 4. Start with a pile of face-up cards in your hand numbered 1 through n. Put the top card onto the bottom of the pile, then put the next card face up on the table. Repeat, putting the top card on the bottom of the pile and then the next card on the table, until eventually all the cards end up sorted on the table. What must have been the original sequence of the cards? If $t(n)$ is the top card of the stack of n, what properties does the sequence of $t(n)$ have as n varies?

Problem 5. How many ways are there to express a given positive integer as the sum of at most two copies of each whole number power of 2?

Problem 6. Does there exist a sequence of positive integers such that its first differences are another copy of the same sequence? How many such sequences are there?

Problem 7. A sequence where deleting every other term leaves a copy of itself?

Problem 8. A sequence of positive integers such that deleting the first occurrence of each number leaves a copy of the original sequence?

Problem 9. A sequence of positive integers such that subtracting one from every term, and then deleting all the zeros, yields a copy of the original sequence?

Problem 10. 0, 1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, ...

Problem 11. 1, 3, 7, 12, 18, 26, 35, 45, 56, 69, 83, 98, 114, ...

Problem 12. 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1, ...

Problem 13. 1, 1, 2, 1, 3, 2, 4, 1, 5, 3, 6, 2, 7, 4, 8, 1, 9, 5, 10, 3, ...

Problem 14. 1, 2, 1, 3, 2, 3, 1, 4, 3, 5, 2, 5, 3, 4, 1, ...