Tilings with 45° Symmetry

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1. Two congruent squares are placed so that the sides of one are at 45° angles to the sides of the other. What are the possibilities for the number of sides of their intersection?

2. A square grid is rotated by 45° about the center of one of its constituent squares. The two grids are superimposed, so that a regular octagon forms at the center.

   (a) Prove that there is no shift by any distance to the right that restores both grids to their original positions.

   (b) Prove that there is no shift in *any* direction that restores both grids to their original positions.

   (c) Prove that no three grid lines meet in a point.

3. Further exploration of the grid described in question 2.

   (a) Draw a horizontal line through the center of the regular octagon. Which types of polygons does it pass through? Can you prove this?

   (b) Repeat for a line through the center of the regular octagon at an angle of 22.5° to the horizontal.

4. [Challenge] Prove that, in any tiling formed by superimposing two congruent square grids at 45° angles, any two tiles with the same number of sides also have the same angles arranged in the same order around the polygon. (For instance, every tile with 4 sides has angles of 45°, 90°, 135°, and 90° in that order.)

5. (a) What are the first three digits after the decimal point in $(1+\sqrt{2})^{2011}$?

   (b) How is this question related to the topic of my talk?