Tilings with 45-degree Symmetry
Dictionary

of Some Important Words

by Evan O'Dorney and Zvezdelina Stankova

Berkeley Math Circle, May 10 2011

(1) **Square Grid**: infinitely many lines drawn in the plane so that by intersecting them one forms the same square all over the plane; in other words, the plane is entirely "tiled" with the same square tile; the tiles do not need to be placed horizontally (or vertically).

(2) **Congruent**: refers to (plane) figures with identical features; for example, two polygons are congruent if by walking along the sides of one polygon one encounters exactly the same lengths of sides and the same sizes of angles as on a walk along the sides of the other polygon; two congruent figures can be obtained from one another by a translation, rotation or reflection (across a line), and any combination of those.

(3) **Congruent Square Grids**: two grids whose defining squares are the same; the two grids, however, need not be oriented the same way, i.e., the squares of one grid may be obtained by rotating (and possibly translating) the squares of the other grid.

(4) **Superimpose**: to lay or place something on or over something else; for instance, to lay one grid over another.

(5) **Translation**: a transformation of the plane which carries all points in the plane along the same chosen direction and at the same chosen distance; for instance, moving every point to the right by 2 cm is a translation.

(6) **Polygon**: a plane figure consisting of several vertices (points) connected by edges, so that by walking along the edges one will pass through each vertex exactly once and will end where one began.

(7) **Quadrilateral**: a figure (polygon) in the plane consisting of 4 vertices (points) and 4 edges (segments). Some famous quadrilaterals are squares, rectangles, trapezoids, rhombuses, and parallelograms.

(8) **Rhombus**: a quadrilateral with 4 equal sides.

(9) **Pentagon**: a polygon with 5 vertices (and 5 edges).

(10) **Hexagon**: a polygon with 6 vertices (and 6 edges).

(11) **Heptagon**: a polygon with 7 vertices (and 7 edges).

(12) **Octagon**: a polygon with 8 vertices (and 8 edges).

(13) **Regular**: refers to a figure (polygon) in the plane whose edges are of the same length, and whose angles are of the same measure. Some famous regular polygons are equilateral triangles, squares, the shape of the Pentagon (in Virginia), and the octagonal shape of STOP signs. Extending the sides of a regular hexagon until they meet again in all possible locations results in the "hexagram"; this is a 6-pointed star figure that can also be viewed as two equilateral triangles offset from each other by a 60° rotation.

(14) **Equilateral**: refers to a figure with equal (in length) sides; most commonly used for "equilateral triangles."

(15) **Equiangular**: refers to a figure with equal (in measure) angles; all regular polygons are equiangular; yet, there are some equiangular polygons whose sides need not be equal, e.g., rectangles that are not squares are equiangular but not regular polygons.

(16) **Aperiodic tiling**: a tiling in which no repeated ("forever") pattern can be found; that is, one cannot cut out a piece of the tiling, and using infinitely many copies of this piece reproduce the whole tiling.