The Euclidean Algorithm from a Geometric Viewpoint

Ted Courant
Berkeley Math Circle
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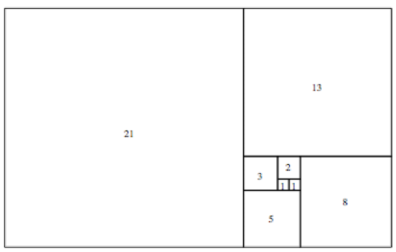
tcourant@bentleyschool.net

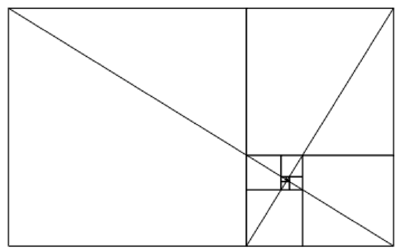
1. Use the Euclidean algorithm to find gcd(97,56); repeat for gcd(99,70).

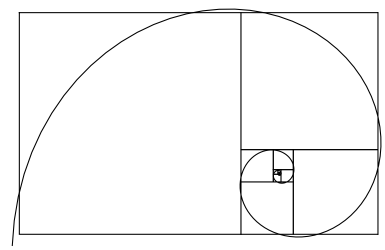
2. Recall that $t_n = \frac{n(n+1)}{2}$ is the n-th triangular number. Show that t_8 is a square; find another square triangular number.

3. Prove that $f_1{}^2 + f_2{}^2 + f_3{}^2 + \cdots + f_n{}^2 = f_n \cdot f_{n+1}$

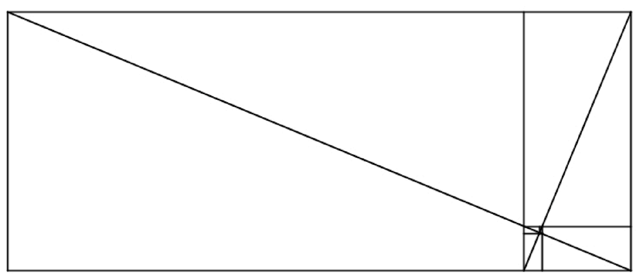
4. Prove that $9\,t_n + 1$ is a triangular number (Fermat), as are $25\,t_n + 3$ and $49\,t_n + 6$ (Euler).



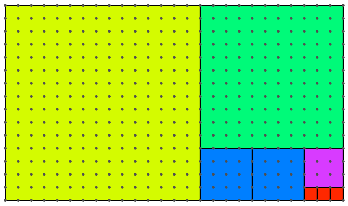




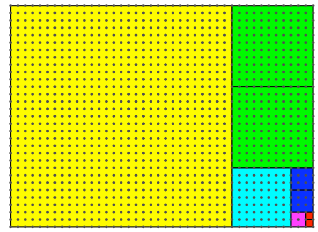
Spiral of 2 by 1 rectangles

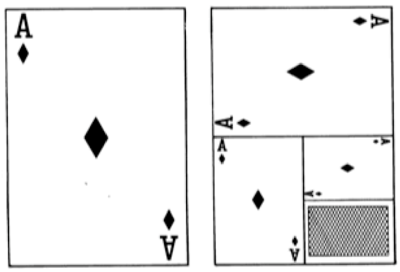


26 × 15

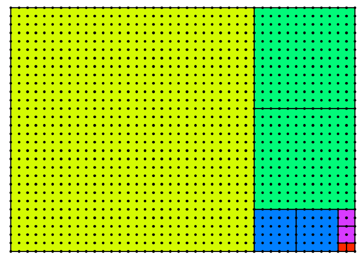


41 × 30





41 × 29



Sources

Excursions in Number Theory by C. Stanley Ogilvy (Dover Book republication (1988))

Fascinating Fractions, by N. M. Beskin (MIR Publishers, The Little Mathematics Library)

Real Numbers and Fascinating Fractions (based on Beskin's book, above: URL: kr.cs.ait.ac.th/~radok/math/mat4/start.htm)

Continued Fractions, by C. D. Olds (New Mathematics Library, MAA

A Problem of Astronomical Proportion, by P. Harvey, The Mathematical Gazette vol 60, number 414, (1976)

A Theorem of Gabriel Lamé, *Mathematical Gems II* by Ross Honsberger, Chapter 7 (Dolciani Mathematical Expositions)