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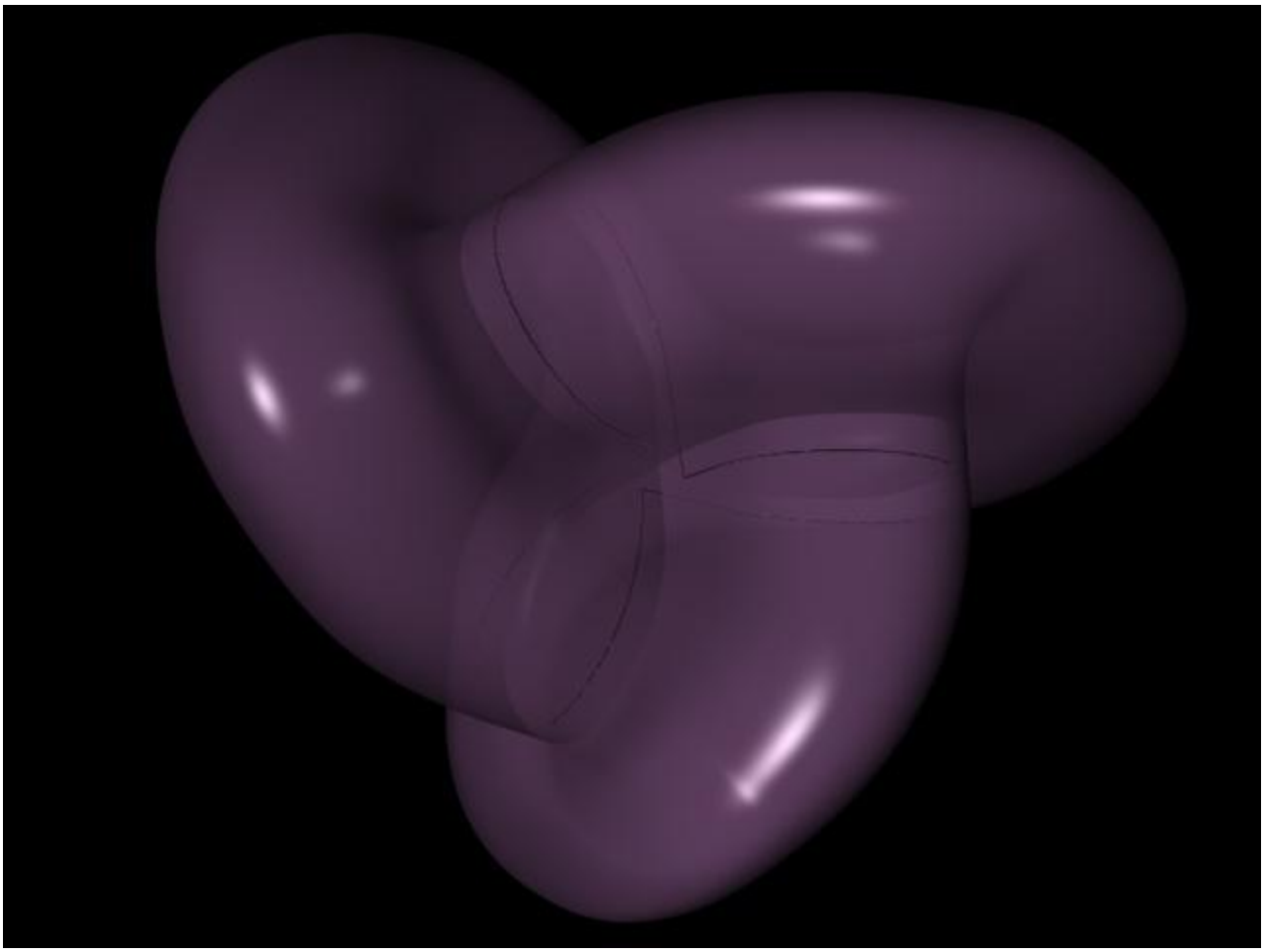
Boy's Surface

Boy's Surface (discovered in 1901 by one Werner Boy) is a single-sided surface with no edges. It is similar to a Klein bottle in this respect, but it is topologically different.
Mathematically speaking, Boy's surface is an immersion of a projective plane in 3-space with no singularities. In English, that means:

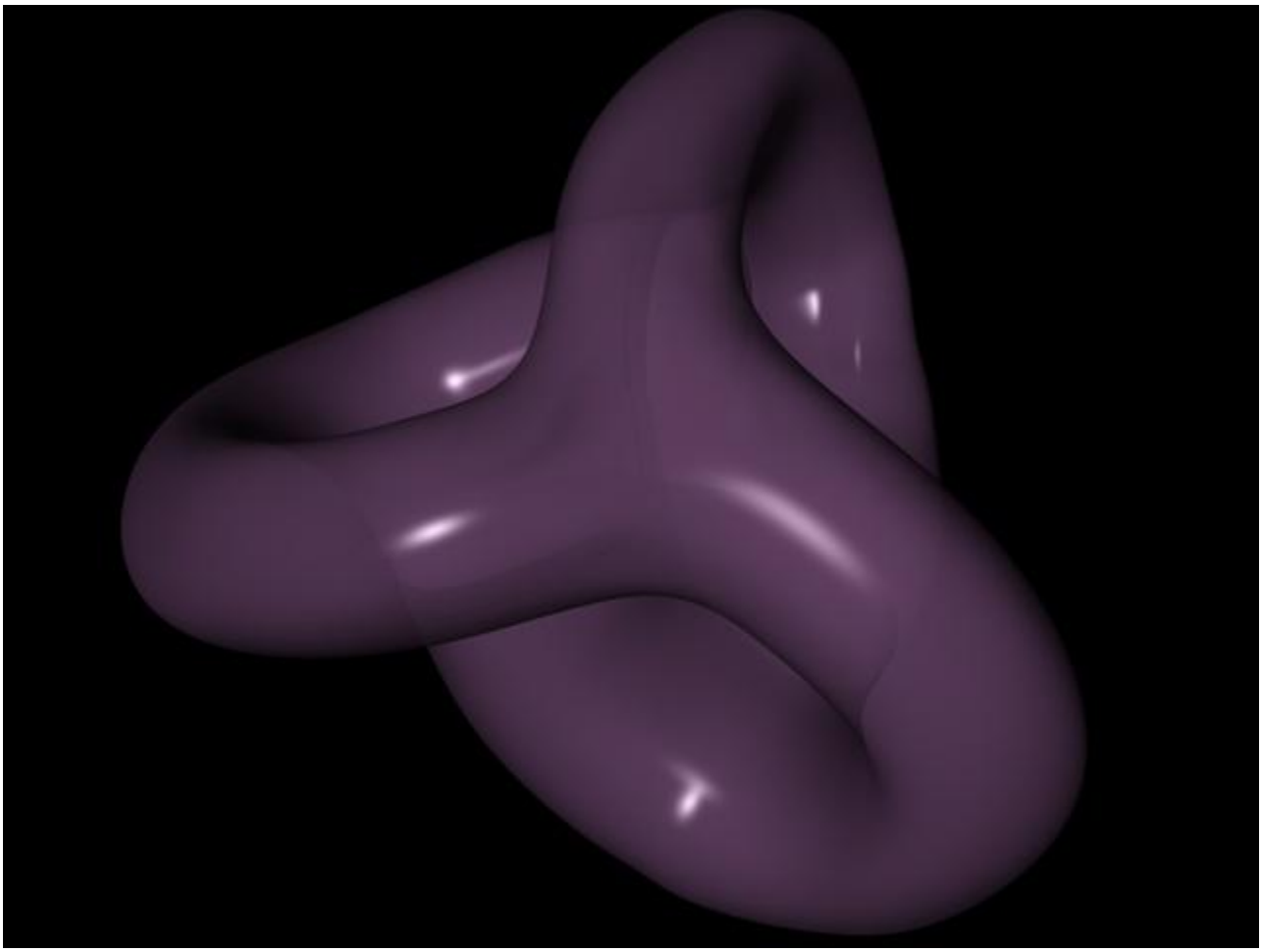
- Start with a disk (made of the infinitely rubbery material topologists like to use for this sort of thing)
- Stretch, bend, and warp the disk until you can glue the edge of the disk to itself in the following way: each section of edge must be attached to the portion of edge directly across from it (on the original, unwarped, disk). There is necessarily a half-twist involved, as in a Möbius strip.
- The surface is allowed to pass through itself, but you can't tear, fold, pinch, or otherwise mutilate the surface.

It's not immediately obvious that you can do such a thing, much less how. For me, a great part of the appeal is that even when you see it in front of you, it's hard to wrap your mind around it.

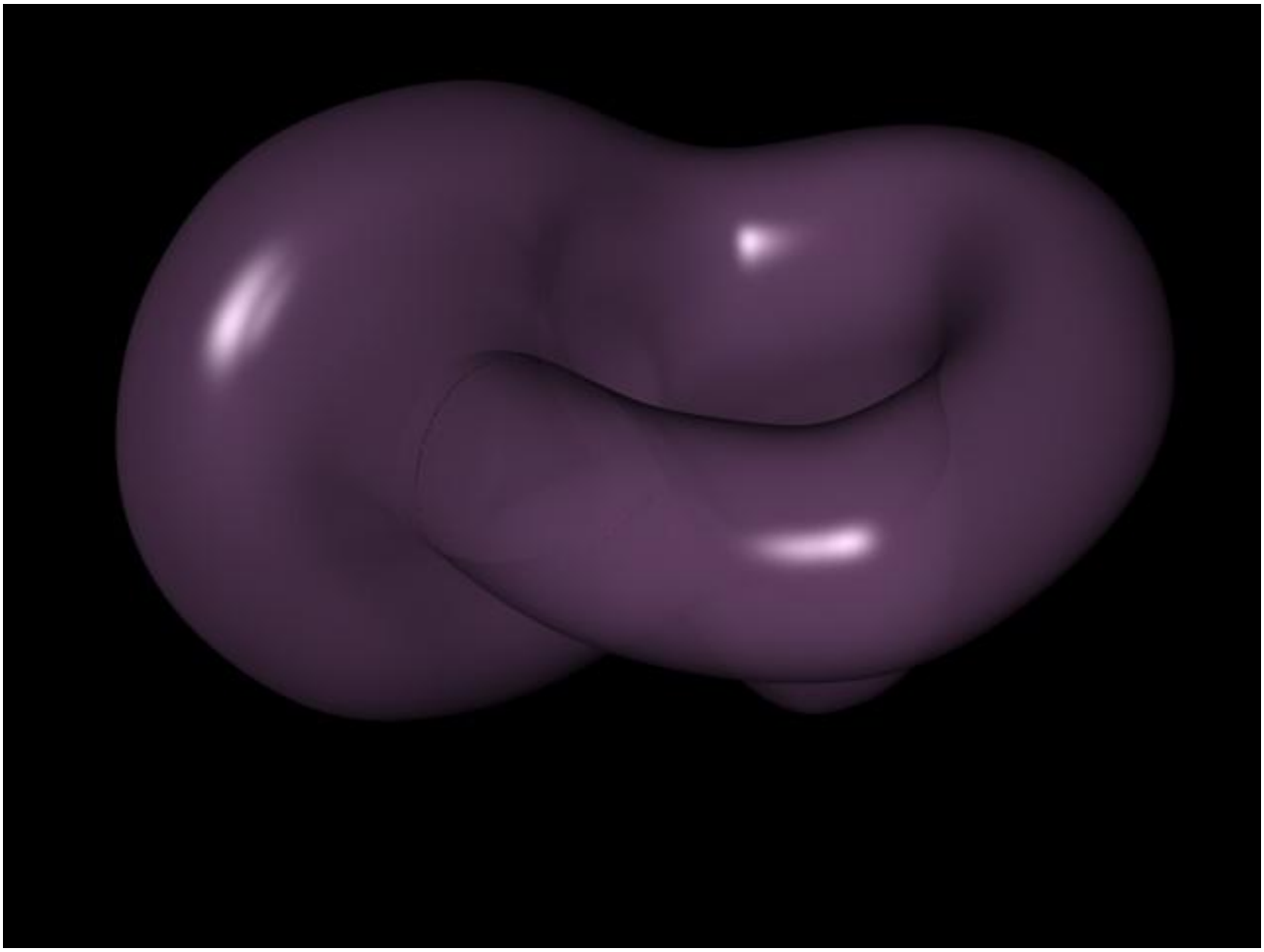
Since a mesh-guided spline is a reasonable approximation to an infinitely rubbery material, that's what I used. It's a free-form Boy's surface: meshed by hand, and thus somewhat irregular, but capable of being further deformed. (most of the other Boy's surfaces available on the web seem to be derived from equations, and are thus exact and symmetric).



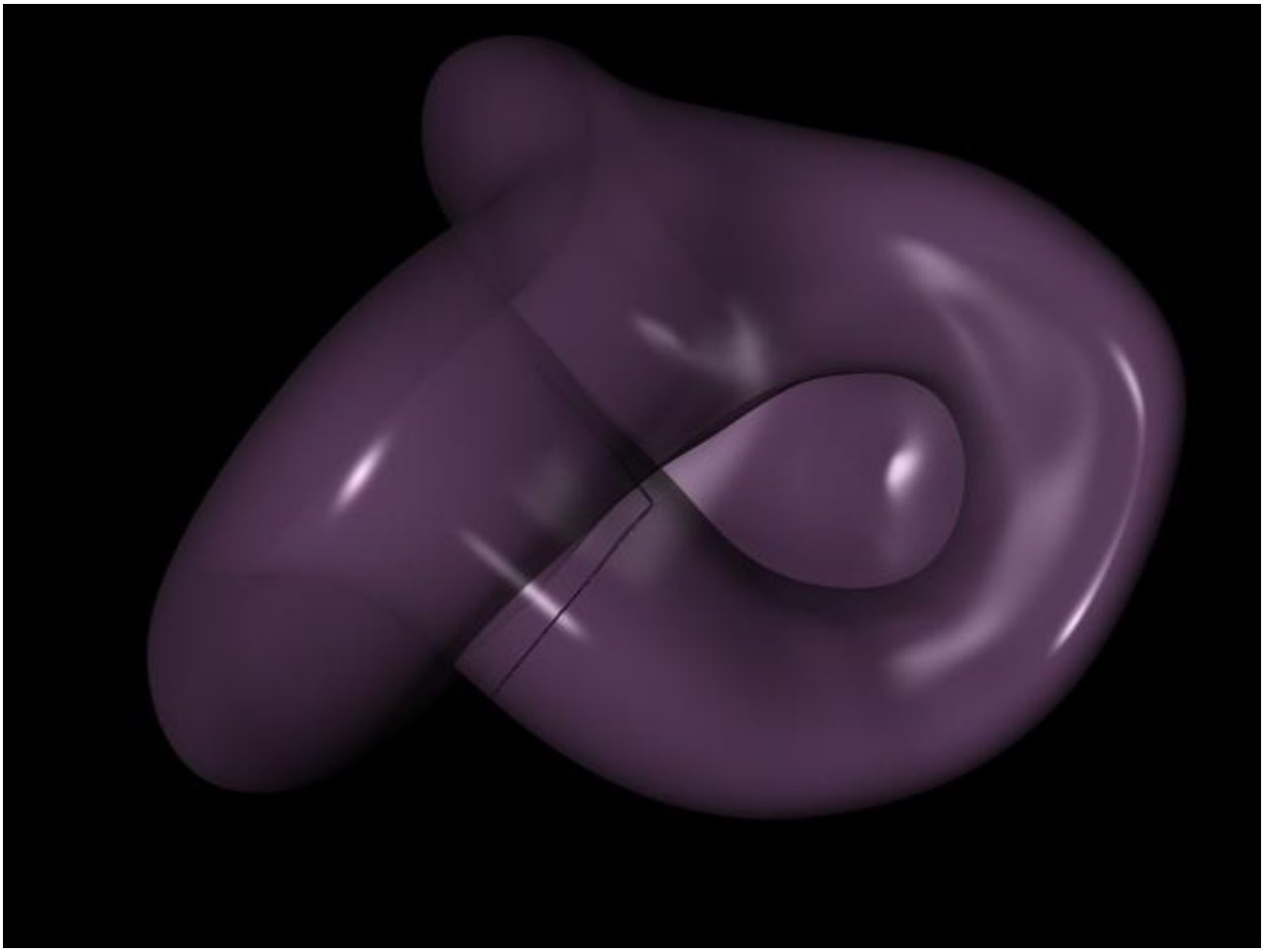
The triple intersection point is directly visible in the center of this view. The curve where the surface intersects itself is shaped like a boat propeller.
Note the black mark that loosely follows the self-intersection curve: since a Boy's surface is single-sided, there's no way to specify a globally consistent surface normal. The mark is where the normal flips from one side to the other.



Here, you're looking at the top of the "main chamber" of the surface. Three "passages" extend from this chamber, each of which end in a "portal" sealed by another part of the surface.



Looking at the end of a passage, from the outside. The curved passage forms the rim of another portal.



Looking down the throat of one of the portals, you also find a side view of the circumscribing passage, which ends in another portal.

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