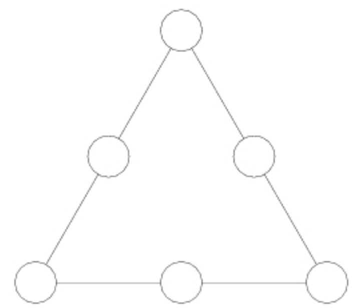
AMC 8 Preparation Problems

1 Algebra, Arithmetic, and Properties of Numbers

1. (1985 AJHSME #24) In a magic triangle, each of the six whole numbers between 10 and 15, inclusive, is placed in one of the circles so that the sum, S, of the three numbers on each side of the triangle is the same. Find the largest possible value for S.



   (A) 36 (B) 37 (C) 38 (D) 39 (E) 40

2. (1987 AJHSME #21) Suppose $n^\star$ means $\dfrac{1}{n}$, the reciprocal of n. For example, $5^\star = \dfrac{1}{5}$. How many of the following statements are true?

   (i) $3^\star + 6^\star = 9^\star$
   (ii) $6^\star - 4^\star = 2^\star$
   (iii) $2^\star \cdot 6^\star = 12^\star$
   (iv) $10^\star \div 2^\star = 5^\star$

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. (1986 AJHSME #25) Which of the following sets of whole numbers has the largest average?

   (A) multiples of 2 between 1 and 101 (B) multiples of 3 between 1 and 101

   (C) multiples of 4 between 1 and 101 (D) multiples of 5 between 1 and 101

   (E) multiples of 6 between 1 and 101

4. (1987 AJHSME #24) A multiple choice examination consists of 20 questions. The scoring is $+5$ for each correct answer, -2 for each incorrect answer, and 0 for each unanswered question. John's score on the examination is 48. What is the maximum number of questions he could have answered correctly?

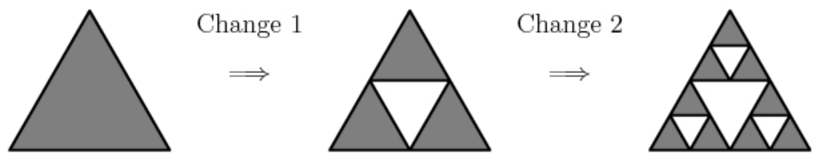
   (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

5. (1988 AJHSME #21) A fifth number, n, is added to the set of numbers $\{3, 6, 9, 10\}$ to make the mean of the set of five numbers equal to its median. The number of possible values for n is:

   (A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

6. (1990 AJHSME #24) Three triangles and a diamond will balance nine circles. One triangle will balance a diamond and a circle. How many circles will balance two diamonds?

7. (1991 AJHSME #25) An equilateral triangle is originally painted black. Each time the triangle is changed, the middle fourth of each black triangle turns white. After 5 changes, what fractional part of the original area of the black triangle remains black?



8. (1991 AJHSME #20) In the addition problem below, each digit has been replaced by a different letter. Different letters represent different digits. Find C.

$$\begin{array}{cccc} & A & B & C \\ & & A & B \\ + & & & A \\ \hline & 3 & 0 & 0 \end{array}$$

9. (2007 AMC 8 #19)Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?

   (A) 2 (B) 64 (C) 79 (D) 96 (E) 131

10. (2007 AMC 8 #18)The product of the two 99-digit numbers

$$303,030,303,\ldots,030,303 \text{ and } 505,050,505,\ldots,050,505$$

   has thousands digit A and units digit B. What is the sum of A and B?

11. (1966 AHSME #46) If the sum of 2 numbers is 1 and their product is 1, what is the sum of their cubes?

12. (1992 MATHCOUNTS) What is the largest integer x for which $1/x$ is larger than $4/49$?

13. (1970 AHSME) Define an operation $\star$ by

$$a \star b = a^b$$

   for all positive numbers a and b. Which of the following is true for all positive a, b, c, n?

   (a) $a \star b = b \star a$
   (b) $a \star (b \star c) = (a \star b) \star c$
   (c) $(a \star b^n) = (a \star n) \star b$
   (d) $(a \star b)^n = a \star (bn)$

14. (2001 AMC 10 #22) In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Five of these numbers are represented by v, w, x, y, z. Find $y + z$.

v	24	w
18	x	y
25	z	21

15. (2001 AMC 12 #19) The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeros, the product of its zeros, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y = P(x)$ is 2, what is b?

16. (2001 AMC 12 #23) A polynomial of degree 4 with leading coefficient 1 and integer coefficients has 2 real zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

    (A) $\frac{1+i\sqrt{11}}{2}$ (B) $\frac{1+i}{2}$ (C) $\frac{1}{2}+i$ (D) $1+\frac{i}{2}$ (E) $\frac{1+i\sqrt{13}}{2}$

17. (2001 AMC 12 #25) Consider sequences of positive real numbers of the form $x, 2000, y, \ldots$, in which every term after the first is 1 less than the product of its 2 immediate neighbors. For how many different values of x does the term 2001 appear somewhere in the sequence?

    (A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

2 Probability and Counting

1. (1985 AJHSME #23) King Middle School has 1200 students. Each student takes 5 classes per day. Each teacher teaches 4 classes. Each class consists of 30 students and 1 teacher. How many teachers are there at King Middle School?

2. (1985 AJHSME #22) For this problem, all telephone numbers are 7-digit whole numbers, and every 7-digit whole number is a possible telephone number except those that begin with 0 or 1. What fraction of telephone numbers begin with 9 and end with 0?

3. (1986 AJHSME #24) The 600 students at King Middle School are divided into three groups of equal size for lunch. Each group has lunch at a different time. A computer randomly assigns each student to one of the three lunch groups. The probability that three friends Al, Bob, and Carol, will be assigned to the same lunch group is approximately:

    (A) $\frac{1}{27}$ (B) $\frac{1}{9}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{3}$

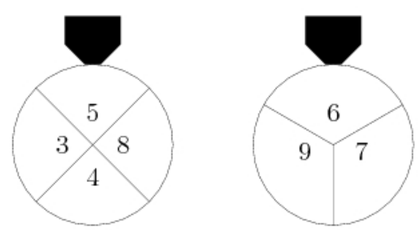
4. (1987 AJHSME #25) Ten balls numbered 1 to 10 are in a jar. Jack reaches into the jar and randomly removes one of the balls. Then Jill reaches into the jar and randomly removes a different ball. The probability that the sum of the two numbers on the balls removed is even is:

    (A) $\frac{4}{9}$ (B) $\frac{9}{19}$ (C) $\frac{1}{2}$ (D) $\frac{10}{19}$ (E) $\frac{5}{9}$

5. (1988 AJHSME #25) A palindrome is a whole number that reads the same forwards and backwards. If one neglects the colon, certain times displayed on a digital watch are palindromes. Three examples are 1:01, 4:44, and 12:21. How many times during a 12-hour period will be palindromes?

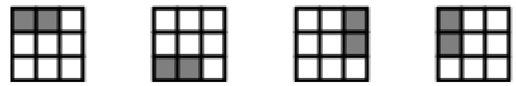
    (A) 57 (B) 60 (C) 63 (D) 90 (E) 93

6. (1989 AJHSME #25) Every time these two wheels are spun, two numbers are selected by the pointers. What is the probability that the sum of the two selected numbers is even?

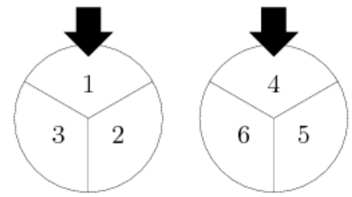


    (A) $\frac{1}{6}$ (B) $\frac{3}{7}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{5}{7}$

7. (1990 AJHSME #25) A 3×3 square is divided into 9 smaller squares. How many different patterns can be made by shading exactly 2 of the 9 squares? Patterns that can be matched by flips and/or turns are *not* considered different. For example, the 4 patterns shown below are all considered to be the same.



8. (1991 AJHSME #22) Every time these two wheels are spun, two numbers are selected by the pointers. These two numbers are then multiplied. What is the probability that this product is an even number?



9. (1992 MAΘ) We are given 5 lines and 2 circles in a plane. What is the maximum number of possible intersection points among these 7 figures?

10. (1992 MATHCOUNTS) How many 3-digit numbers are palindromes?

11. (Mandelbrot #3) Find the sum of all 4-digit palindromes.

12. (Mandelbrot #2) If $\frac{a!}{b!}$ is a multiple of 4 but not a multiple of 8, what is the maximum value of $a - b$?

13. (1996 AJHSME #25) A point is chosen at random from within a circular region. Find the probability that the point is closer to the center of the region than it is to the boundary of the region.

14. (2001 AMC 10 #23) A box contains exactly 5 chips, 3 red and 2 white. Chips are randomly removed 1 at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

15. (2005 AMC 8 #24) A certain calculator has only two keys: [+1] and [×2]. When you press one of the keys, the calculator automatically displays the result. For example, if the calculator originally displayed "9" and you pressed [+1] it would display "10." If you then pressed [×2], it would display "20." Starting with display "1," what is the fewest number of keystrokes you would need to reach "200"?

16. (2001 AMC 12 #16) A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

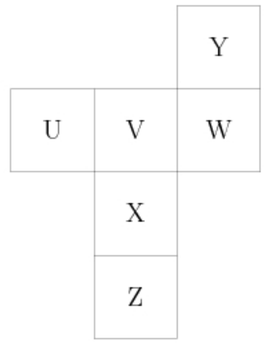
17. (2001 AIME I #14) A mail carrier delivers mail to the 19 houses on the east side of Elm Street. The carrier notices that no 2 adjacent houses ever get mail on the same day, but that there are never more than 2 houses in a row that get no mail on the same day. How many different patterns of mail delivery are possible?

18. (2001 AIME I #15) The numbers $1, 2, 3, 4, 5, 6, 7, 8$ are randomly written on the faces of a regular octahedron so that each face contains a different number. The probability that no 2 consecutive numbers, where 8 and 1 are considered to be consecutive, are written on faces that share an edge is m/n, where m and n are relatively prime positive integers. Find $m + n$.

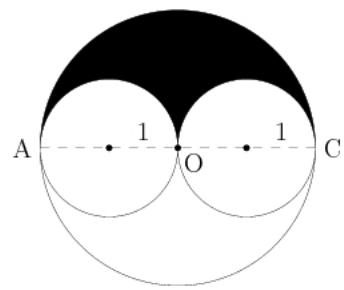
19. (2001 AIME II #9) Each unit square of a 3×3 unit-square grid is to be colored either blue or red. For each square, either color is equally likely to be used. The probability of obtaining a grid that does not have a 2×2 red square is m/n, where m and n are relatively prime positive integers. Find $m + n$.

3 Geometry and Spatial Visualization

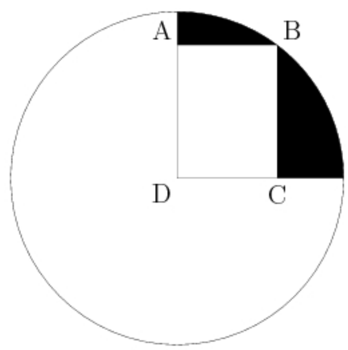
1. (1985 AJHSME #11) A piece of paper containing six joined squares labeled as shown in the diagram below is folded along the edges of the squares to form a cube. What is the label of the face opposite X?



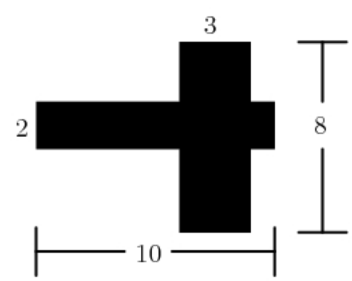
2. (1986 AJHSME #23) The large circle has diameter AC. The two small circles have their centers on AC and just touch at O, the center of the large circle. If each small circle has radius 1, what is the value of the ratio of the area of the shaded region to the area of one of the small circles?



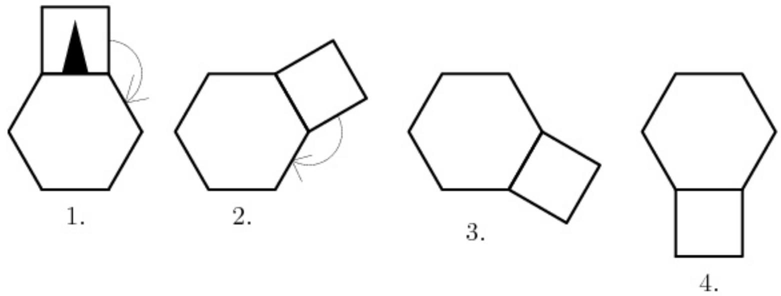
3. (1987 AJHSME #22) $ABCD$ is a rectangle, D is the center of the circle, and B is on the circle. If $AD = 4$ and $CD = 3$, find the area of the shaded region.



4. (1988 AJHSME #17) Find the area (in square units) of the shaded region formed by the two intersecting perpendicular rectangles shown below.

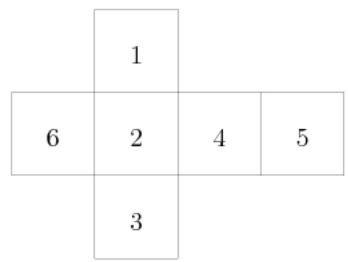


5. (1988 AJHSME #24) The square in the first diagram "rolls" clockwise around the fixed regular hexagon until it reaches the bottom. In which position will the solid triangle be in diagram 4?

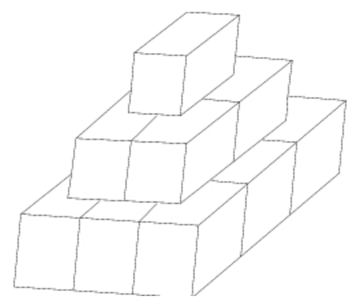


(A)  (B)  (C)  (D)  (E) 

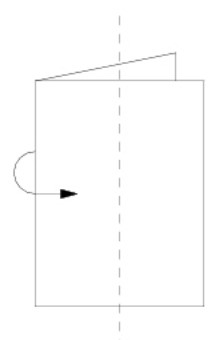
6. (1989 AJHSME #20) The figure may be folded along the lines shown to form a number cube. Three number faces come together at each corner of the cube. What is the largest sum of three numbers whose faces come together at a corner?



7. (1989 AJHSME #23) An artist has 14 cubes, each with an edge of 1 meter. She stands them on the ground to form a sculpture as shown. She then paints the exposed surface of the sculpture. How many square meters does she paint?

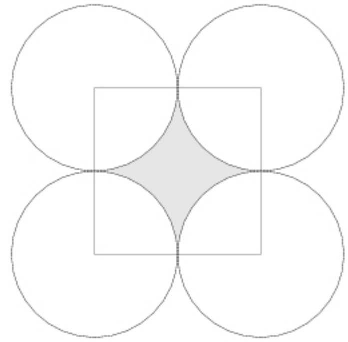


8. (1989 AJHSME #24) Suppose a square piece of paper is folded in half vertically. The folded paper is then cut in half along the dashed line. Three rectangles are formed - a large one and two small ones. What is the ratio of the perimeter of one of the small rectangles to the perimeter of the large rectangle?



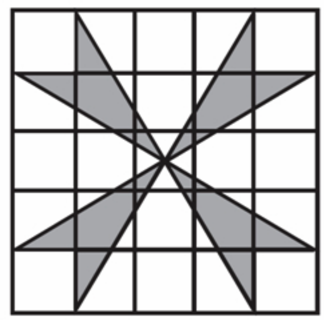
9. (1991 AJHSME #24) A cube of edge 3 cm is cut into N smaller cubes, not all the same size. If the edge of each of the smaller cubes is a whole number of centimeters, find N.

10. (1992 AJHSME #24) Four circles of radius 3 are arranged as shown. Their centers are the vertices of a square. Find the area of the shaded region.

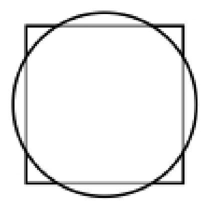


11. (2001 AMC 8 #23) Points R, S, T are vertices of an equilateral triangle, and points X, Y, Z are midpoints of its sides. How many noncongruent triangles can be drawn using any 3 of the 6 points as vertices?

12. (2007 AMC 8 #23) What is the area of the shaded pinwheel shown in the 5×5 grid?



13. (2005 AMC 8 #25) A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?



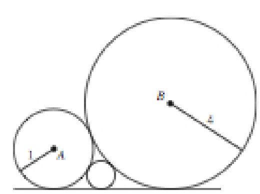
14. (2005 AMC 8 #23) Isosceles right triangle ABC encloses a semicircle of area 2π. The circle has its center O on hypotenuse AB and is tangent to sides AC and BC. What is the area of triangle ABC?

15. (2001 AMC 10 #20) A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 2000. What is the length of each side of the octagon?

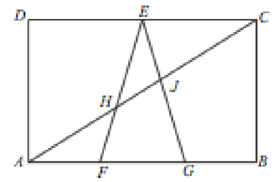
16. (2001 AMC 10 #21) A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Find the radius of the cylinder.

17. (2001 AMC 10 #24) In trapezoid $ABCD$, AB and CD are perpendicular to AD, with $AB + CD = BC$, $AB < CD$, and $AD = 7$. What is $AB \cdot CD$?

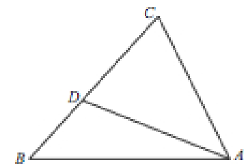
18. (2001 AMC 12 #18) A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first 2 and to 1 of their common external tangents, as shown. Find the radius of the third circle.



19. (2001 AMC 12 #22) In rectangle $ABCD$, points F and G lie on AB so that $AF = FG = GB$ and E is the midpoint of DC. Also, AC intersects EF at H and EG at J. The area of rectangle $ABCD$ is 70. Find the area of triangle EHJ.



20. (2001 AMC 12 #24) In triangle ABC, the measure of angle ABC is 45 degrees. Point D is on BC so that $2 \cdot BD = CD$ and the measure of angle DAB is 15 degrees. Find the measure of angle ACB.



21. (2001 AIME I #12) A sphere is inscribed in the tetrahedron whose vertices are $A = (6, 0, 0)$, $B = (0, 4, 0)$, $C = (0, 0, 2)$, and $D = (0, 0, 0)$. The radius of the sphere is m/n, where m and n are relatively prime positive integers. Find $m + n$.

22. (2001 AIME I #13) In a certain circle, the chord of a d-degree arc is 22 centimeters longer than the chord of a $3d$-degree arc, where $d < 120$. The length of the chord of a $3d$-degree arc is $-m + \sqrt{n}$ centimeters, where m and n are positive integers. Find $m + n$.