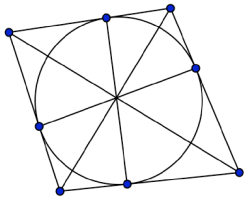
Geometric Transformations with an Introduction to Techniques from Projective Geometry
Part II

Ted Courant
Berkeley Math Circle
September 29, 2009

Questions

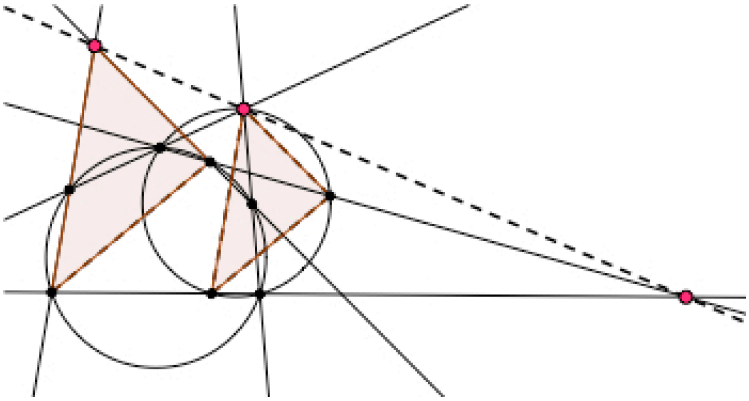
Given a quadrilateral with an inscribed circle, as shown, prove that the diagonals and the chords connecting the opposite points of tangency are concurrent.



A straight irrigation ditch leads away from a storage tank in the middle of a level field; the tank is too big to see over or around. How can you continue the straight path of the ditch on the other side of the tank using only marker pegs driven into the ground?

Exercises (continued)

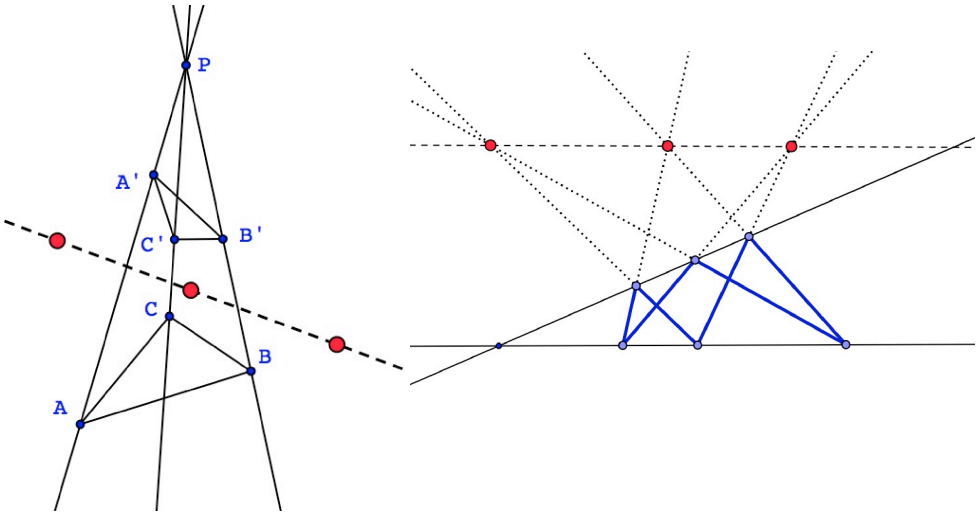
1. Given two circles, prove that the locus of points with equal Powers to the two circles is a line.
2. Prove that $A'B' = r^2 \dfrac{AB}{OA\ OB}$ where O is the center of inversion, and r is the radius of circle O.
3. Prove that orthogonal circles remain fixed under inversion by each other.
4. If two circles are orthogonal, then the diameter of one circle cuts the other in a pair of points which are inverses of one another. Moreover, this is true of any radial line of one circle which cuts the other circle in two points.
5. Given three concurrent circles, how many circles may be constructed tangent to all three?
6. Prove that any three disjoint circles can be inverted into circles whose centers are collinear.
7. Use the picture below to provide an easy proof of Pascal's theorem: If a hexagon is inscribed in a circle, then the intersections of the three pairs of opposite sides are concurrent.



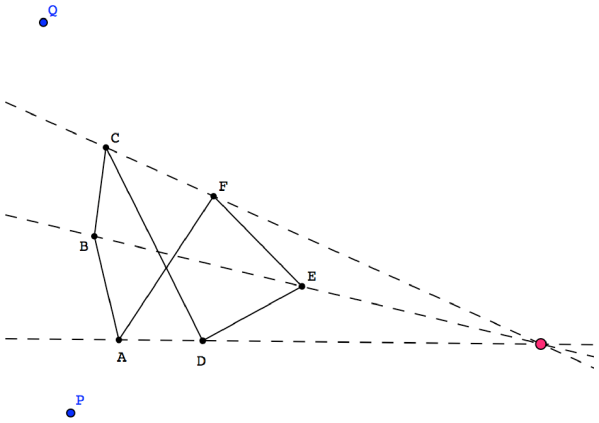
Projective Geometry

1. **Desargue's Theorem** Prove that if two triangles are in perspective from a point, the three intersections of their corresponding sides are concurrent. Prove the theorem in space first, then attempt the plane version.

2. **Pappus' Theorem** If the vertices of a hexagon lie alternately on two lines, then the three intersections of the pairs of opposite sides are concurrent.



3. **Brianchon's Theorem** If the sides of a hexagon pass alternately through two fixed points P and Q, then the three diagonals joining opposite sides of the hexagon are concurrent.



Suggested Reading

Excursions in Geometry, C. Stanley Ogilvy (Dover Books)

Introduction to Geometry, Coxeter (Wiley and Sons)

Geometry: A Comprehensive Course, Dan Pedoe (Dover Books)

Reflections on the Arbelos, Harold P. Boas, American Mathematical Monthly **113** (March 2003)
(http://www.math.tamu.edu/harold.boas/preprints/arbelos.pdf)

Modern Geometries, James Smart (Brooks/Cole)

tcourant@bentleyschool.net