1. a. Prove that it's possible to cover an 8×8 chessboard with 2×1 rectangular tiles.
   b. Assume now that we remove one corner of the board. Is it still possible to do this? Prove your statement.
   c. What if we remove 2 opposite corners of the board?

2. Prove that the set of prime numbers is infinite. (This proof, going back to Euclid, remains one of the classic examples of elegance of mathematics.)

3. Let $n \geq 2$ be a positive integer such that $(n-1)!+1$ is divisible by n. Prove that n is prime.

4. Prove that $\sqrt{2}$ is not a rational number. Can you try a generalization of this statement?

5. a.Prove that if a, b, c are odd integers then the equation $ax^2 + bx + c = 0$ can not have a rational root.
   b. Prove that if a, b, c are odd integers the equation $ax^n + bx + c = 0$ can not have a rational root.

6. a.Let a, b, c be positive integers such that $a^2+b^2 = c^2$ and their greatest common divisor is 1. Prove that c is odd.
   b. Prove that the equation $a^2+b^2 = 3c^2$ doesn't have any integer solutions.

7. Prove that if 2 angles of a triangle are equal then the opposite sides are also equal.

Pigeonhole principle:
a. If you put $n+1$ pigeons into n holes then at least one hole will contain more than one pigeon.
b. If you put $kn+1$ pigeons into n holes then at least one hole will contain more than k pigeons.

8. Prove that if we pick 11 integers from the set $\{1, 2, 3, \ldots 20\}$ then at least 2 of them have to be consecutive.
9. Prove that among any $n + 1$ numbers from a set of $2n$ consecutive integers there are 2 whose difference is n.
10. Let 5 points be on a sphere. Prove that there is a closed hemisphere which contains 4 of them.
11. Prove that out of any 5 points which lie inside an equilateral triangle of side-length 2 one can find 2 at distance at most 1.
12. A party is attended by $n \geq 2$ people, some of which are friends, some of which are not. Prove that there are 2 people with the same number of friends. (Here we assume that if A is friend of B than B is also a friend of A).
13. Prove that if a, b are coprime positive integers there exist nonzero integers x, y such that $ax - by = 1$. Formulate the converse of this statement. Is it true?
14. a. Assume all points in the plane are colored red or blue. Prove that there are 2 points of the same color $1ft$ apart.
    b. Assume all points in the plane are colored red, blue and green. Prove that there are 2 points of the same color $1ft$ apart.
    c. Try to think of a coloring of the plane in 7 colors such that there are no 2 points of the same color who are $1ft$ apart.
15. Assume all points in the plane are colored red or blue. Prove that there is a rectangle whose vertices are the same color.