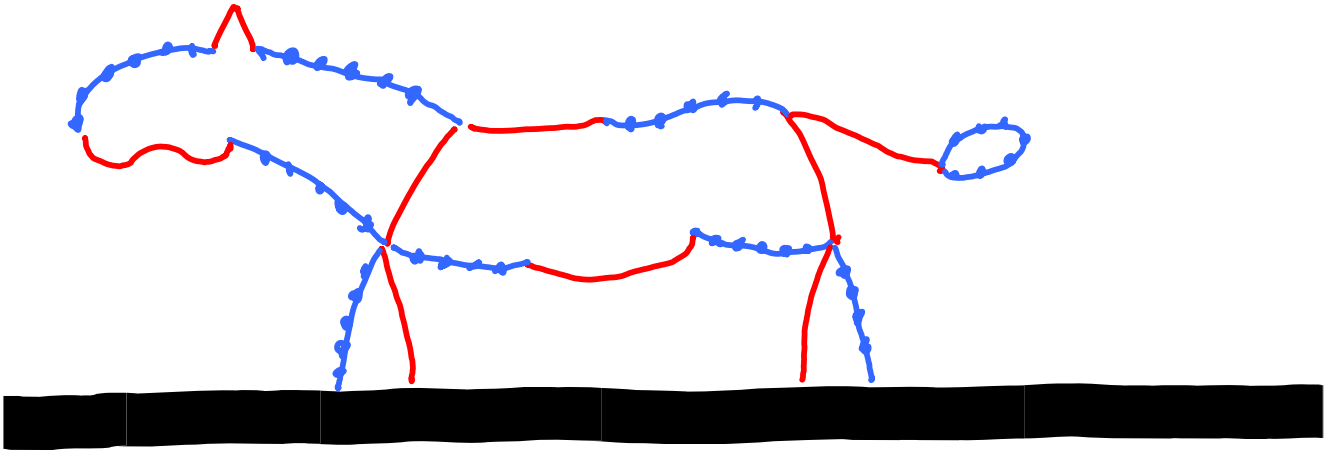
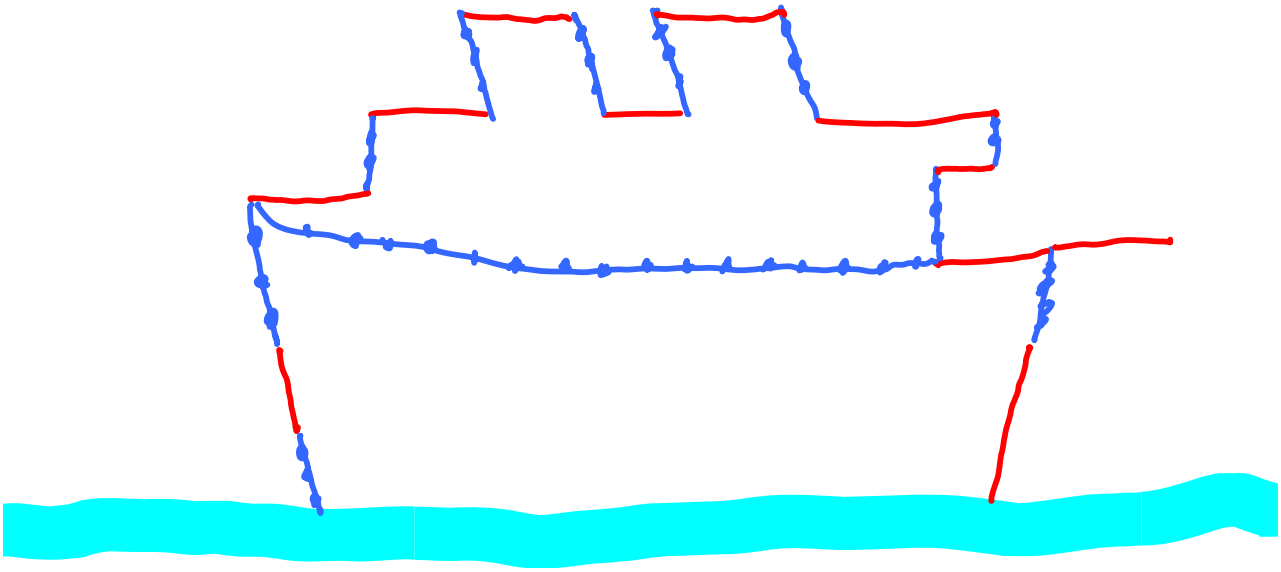
It's all fun and games.

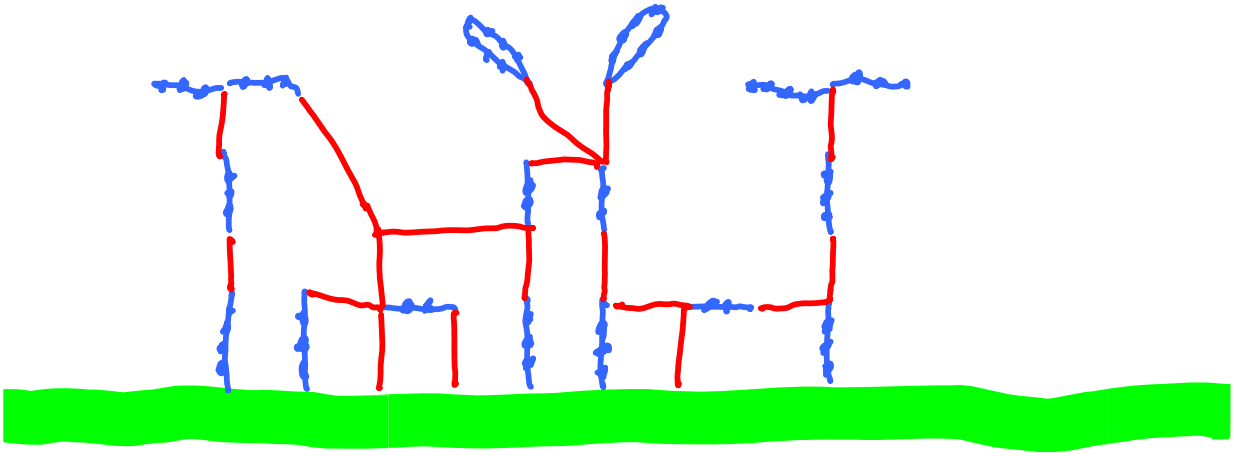
See: Berlekamp, Conway and Guy, Winning Ways for your mathematical plays

Hackenbush:

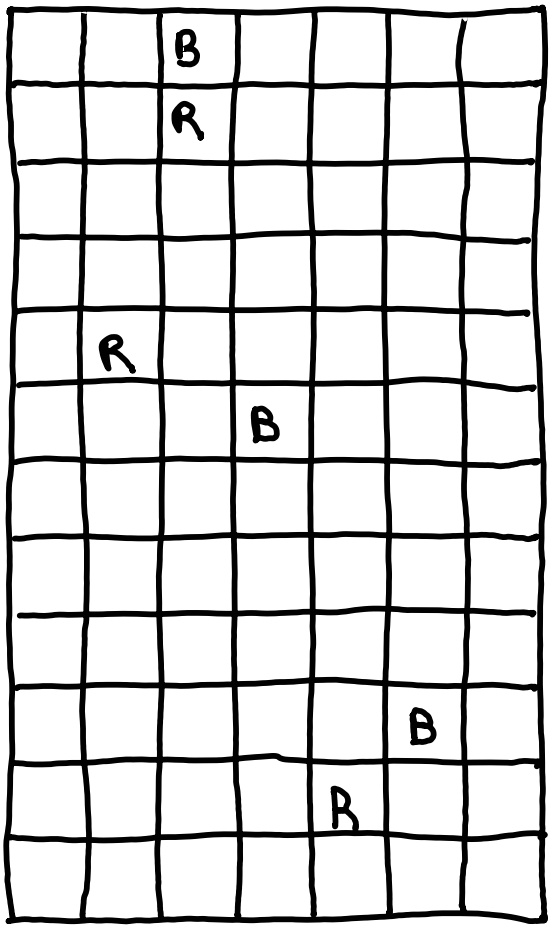
Players take turns erasing sticks from a drawing or figure. Each player has a color and can only erase sticks of that color. Whenever a collection of sticks is no longer connected to the ground it is erased.







Ski jumps:
In this game Blue can move any man to the left, porvided there is no man in his way. If a skier has not been jumped he may jump over a Red skier immediately below provided the space is empty. Red can move in the same way but he moves to the right and down. The last player to move wins.

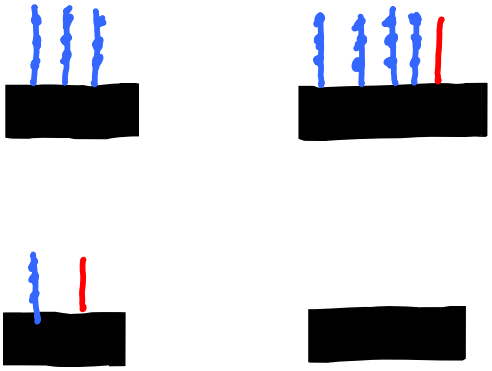


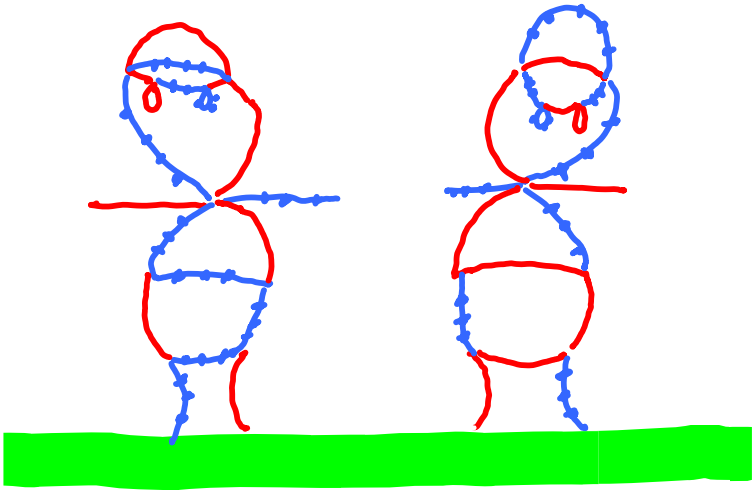
Some natural questions are:

Who will will?

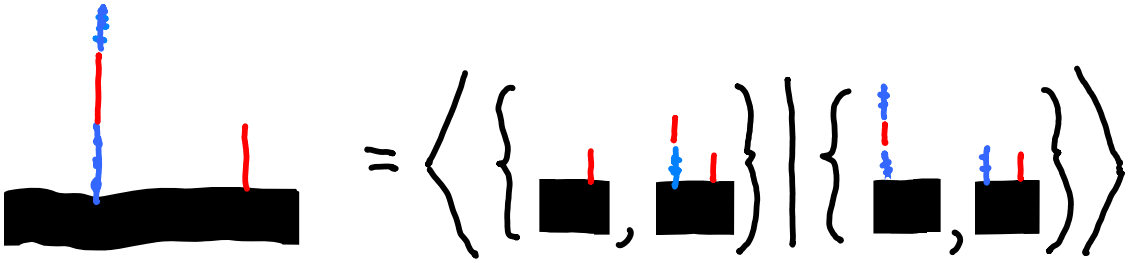
By how many moves?

We start with some easy games of hackenbush.





To keep track of a game we can write all possible moves that blue can make and all possible moves that red (red on the right) can make at the start fo the game. For example:

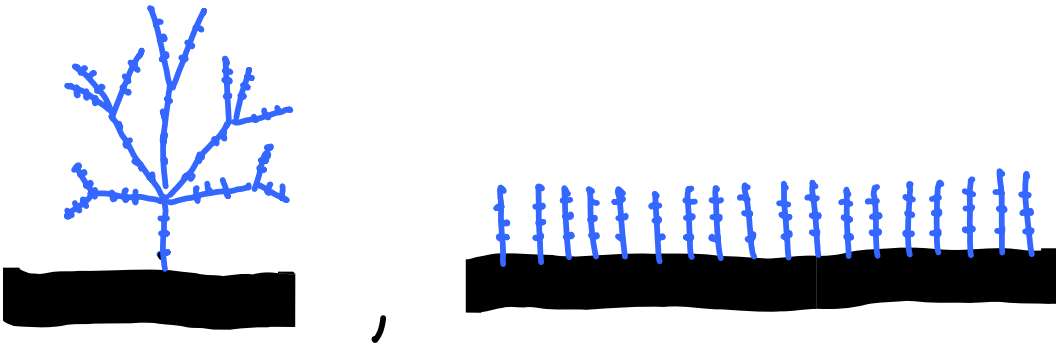


It is possible to add two games: Just put them side by side, then each player make make one move on either game when it is their turn. It is also possible to take the negation of a game (the players just trade sides).
A game is:
positive is blue can always win,
negative if red can always win,
zero if the first person to move always loses
Here we assume that players always play the best that they can.

Problems:

1. Replace each of the game boards in the list of possible moves with its own list of moves. Repeat this process until there are no more moves in any of the resulting positions.

2. Show that the following games are equal:



3. Make up simple hackenbush games representing each of the following numbers.

$0 = \langle \{\,\} | \{\,\} \rangle$, $1 = \langle 0 | \{\,\} \rangle$, $2 = \langle 1 | \{\,\} \rangle$, ...

$-1 = \langle \{\,\} | 0 \rangle$, $-2 = \langle \{\,\} | -1 \rangle$, ...

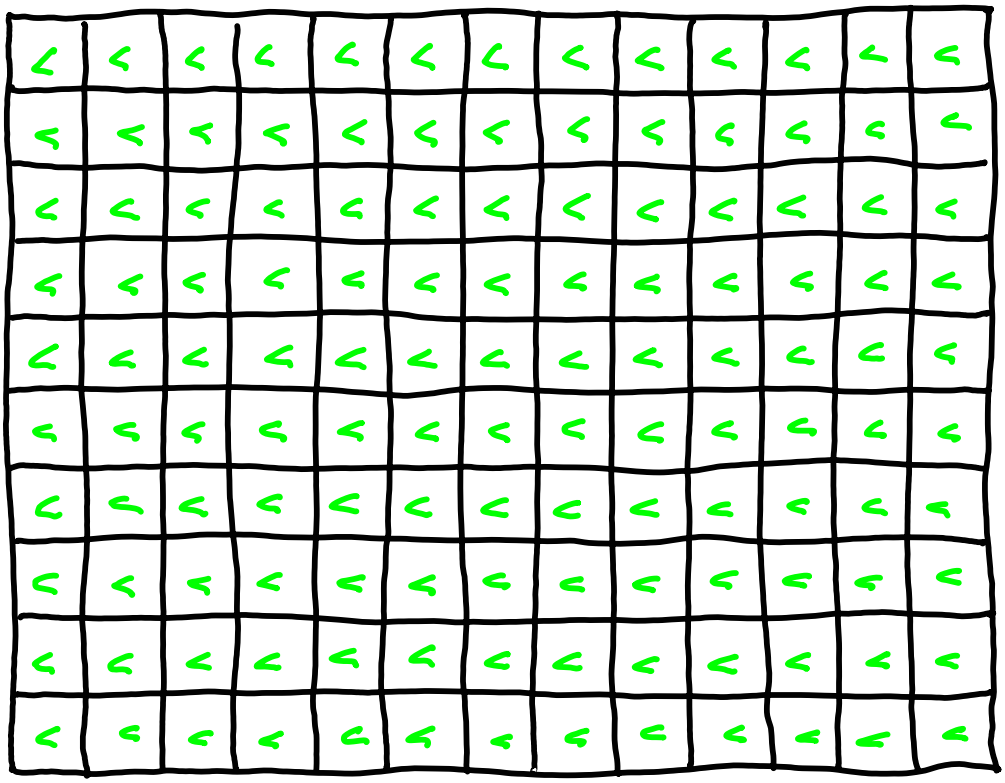
4. Give alternative hackenbush games for at least two of the above numbers, and write out the entire move list for your alternative games.

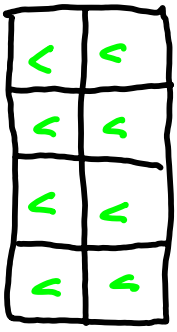
Game parts

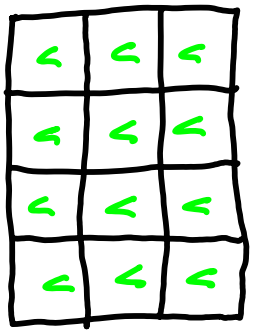
If you cut two pieces of paper into four equal parts, how much paper is each part?
What other ways are there to make parts of the same size?
Can we have parts of games, i.e. win by 1/2 of a move? Before answering these, let's play a new game.

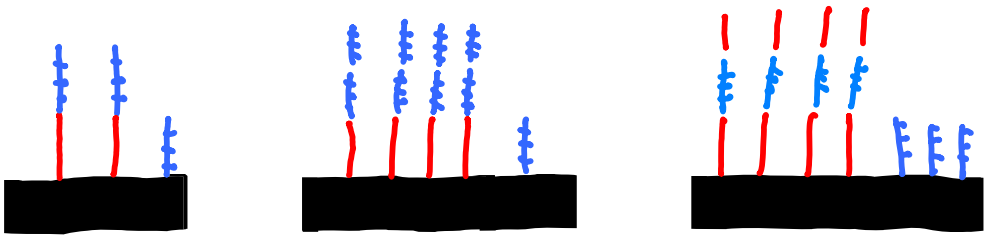
Cut cake

The top of a cake is marked with squares. Players take turns cutting the cake all the way across just one contiguous rectangle (red, left and Right blue up and down) until there are no more legal cuts. The last person to cut wins.

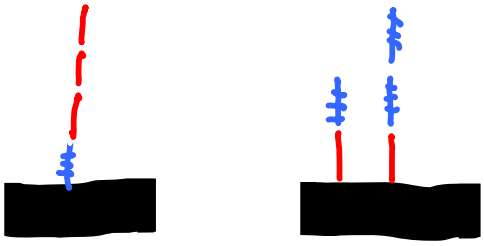


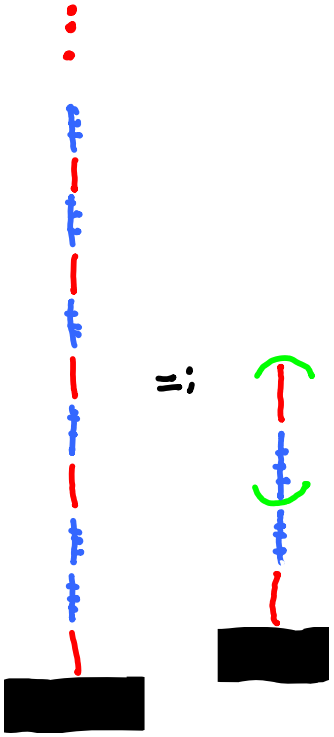






Who wins each of the above if red goes first? if blue goes first?
What is the value of each?





What is the value of each game here?

If the one stick has length 1, how long are each of the rest?