# SUMMING UP “GAUSS-FASHION”


**Versions of the Gauss Schoolroom Anecdote**
http://www.sigmaxi.org/amscionline/gauss-snippets.html

The story dates from Gauss's schooldays, when he was about ten years old. At the first meeting of the arithmetic class, Master Büttner asked the pupils to write down the numbers from 1 through 100 and add them. It was the custom that the pupils lay their slates, with their answers thereon, on the master's desk upon completion of the problem. Master Büttner had scarcely finished stating the exercise when young Gauss flung his slate on the desk. The other pupils toiled on for the rest of the hour while Gauss sat with folded hands under the scornful and sarcastic gaze of the master. At the conclusion of the period, Master Büttner looked over the slates and discovered that Gauss alone had the correct answer!

And thus, the sum 1+2+3+…+100 is rumored to have been figured out by the primary school pupil named **Johann Carl Friedrich Gauss (1777-1855)** while attending his boring arithmetic class. There are hundreds of versions of this famous Gauss schoolroom story, probably the most frequently told math anecdote. The sum itself is the simplest example of the so-called *arithmetic progression*. And this was just the beginning for Gauss.

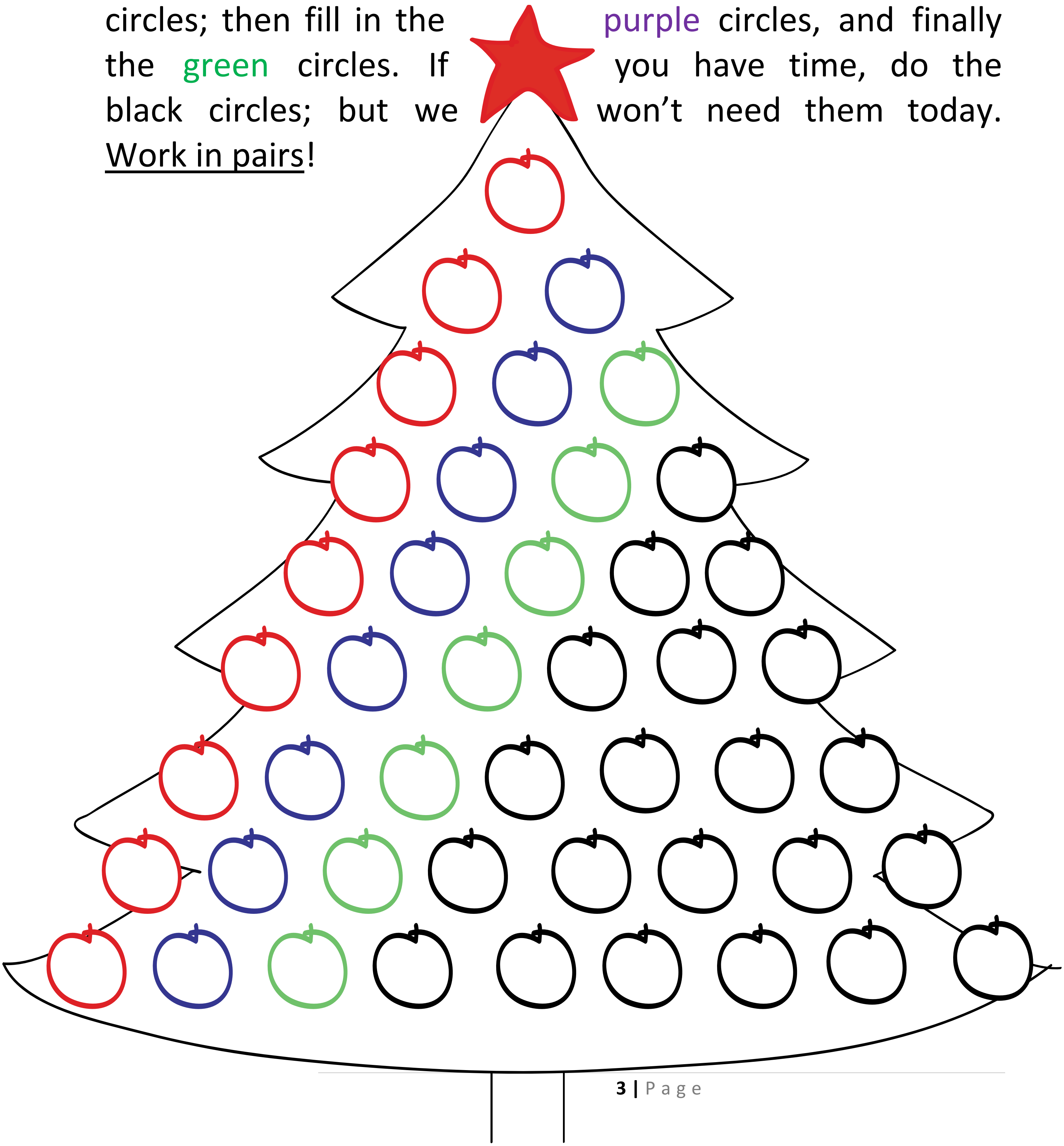
Due to his enormous influence and contributions to many mathematical and science fields, some consider Gauss the **``greatest mathematician since antiquity''**; some call him simply **``the Prince of Mathematicians''**; while he himself refers to mathematics royally as **``the queen of the sciences.''**

**Main Question 1:** How did young Gauss sum up all numbers from 1 to 100:

$$1 + 2 + 3 + 4 + 5 + \ldots + 97 + 98 + 99 + 100$$

without writing much on his tablet, correctly, and so fast?

**Question 2 (Group 1):** Does this Holiday Tree resemble the Pascal Triangle? Fill in the numbers in this Holiday Tree according to the Pascal Triangle rules. Start with the red circles; then fill in the purple circles, and finally the green circles. If you have time, do the black circles; but we won't need them today. Work in pairs!



**<u>Question 3 (Group 2)</u>:** How do mathematicians attack hard problems? They try first simple cases. The boy Gauss could have started by adding up fewer numbers!

Calculate the following sums. Work in pairs and check each other's answers. Think: Is there a shortcut for finding the answers to the next sum knowing the answer to the previous sum?

1 = ______

1 + 2 = ______

1 + 2 + 3 = ______

1 + 2 + 3 + 4 = ______

1 + 2 + 3 + 4 + 5 = ______

1 + 2 + 3 + 4 + 5 + 6 = ______

1 + 2 + 3 + 4 + 5 + 6 + 7 = ______

**Question 4 (Both groups):** Do you see the answers to the sum in Question 3 somewhere in the holiday tree? Where? Do you think this is a coincidence or is this pattern going to go on forever?

Check your conjecture for

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = \_\_\_\_\_\_?$$

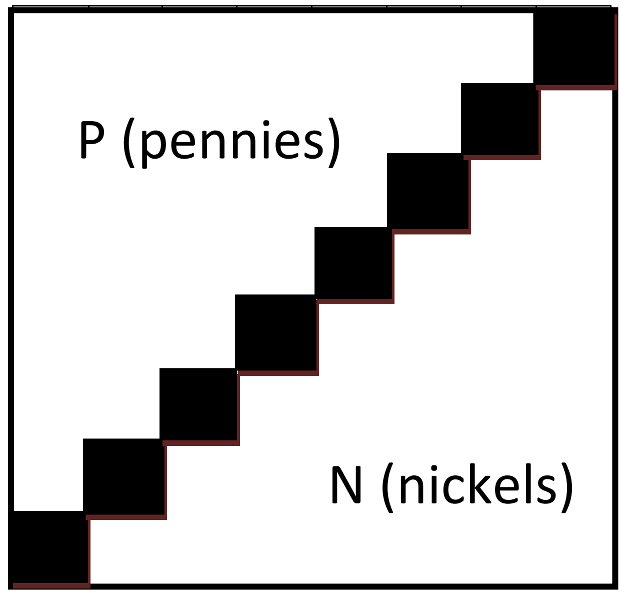
Is the answer in the Holiday Tree? Why do you think it is there?

**Question 5 (Both groups):** Do you think Gauss used the Holiday Tree/Pascal Triangle to figure out

$$1 + 2 + 3 + 4 + 5 + \ldots + 97 + 98 + 99 + 100 = ?$$

**<u>Question 6 (everyone)</u>:**

Let's try a geometric approach. Cover your chessboards as follows:




(a) Put <u>quarters</u> along the diagonal that starts from left bottom corner and goes to the right top corner.

(b) Put a <u>penny</u> in each cells above the diagonal.

(c) Put a <u>nickel</u> in each cells below the diagonal.

(d) Write down how many coins you used:

- Quarters: Q = ______
- Pennies: P = ______
- Nickels: N = ______
- Total: T = ______

$$Q + P + N = T$$

(e) What do you notice about P and N?

(f) Can we figure out T <u>without</u> counting the coins?

(g) Can we figure out P and N <u>without</u> counting the coins?

## Answers to Question 6:

(e) P = N
(f) T = $8^2$ = 64: numbers of squares is a chessboard
(g) The equation to determine P and N is:

$$Q + P + N = T \rightarrow Q + P + P = T$$

Q = 8 = number of diagonal cells
T = 64 = total number of cells

$$\rightarrow 8 + P + P = 64 \rightarrow 8 + 2P = 64$$

$$\rightarrow 2P = 64 - 8 = 56 \rightarrow P = 56 : 2 = 28.$$

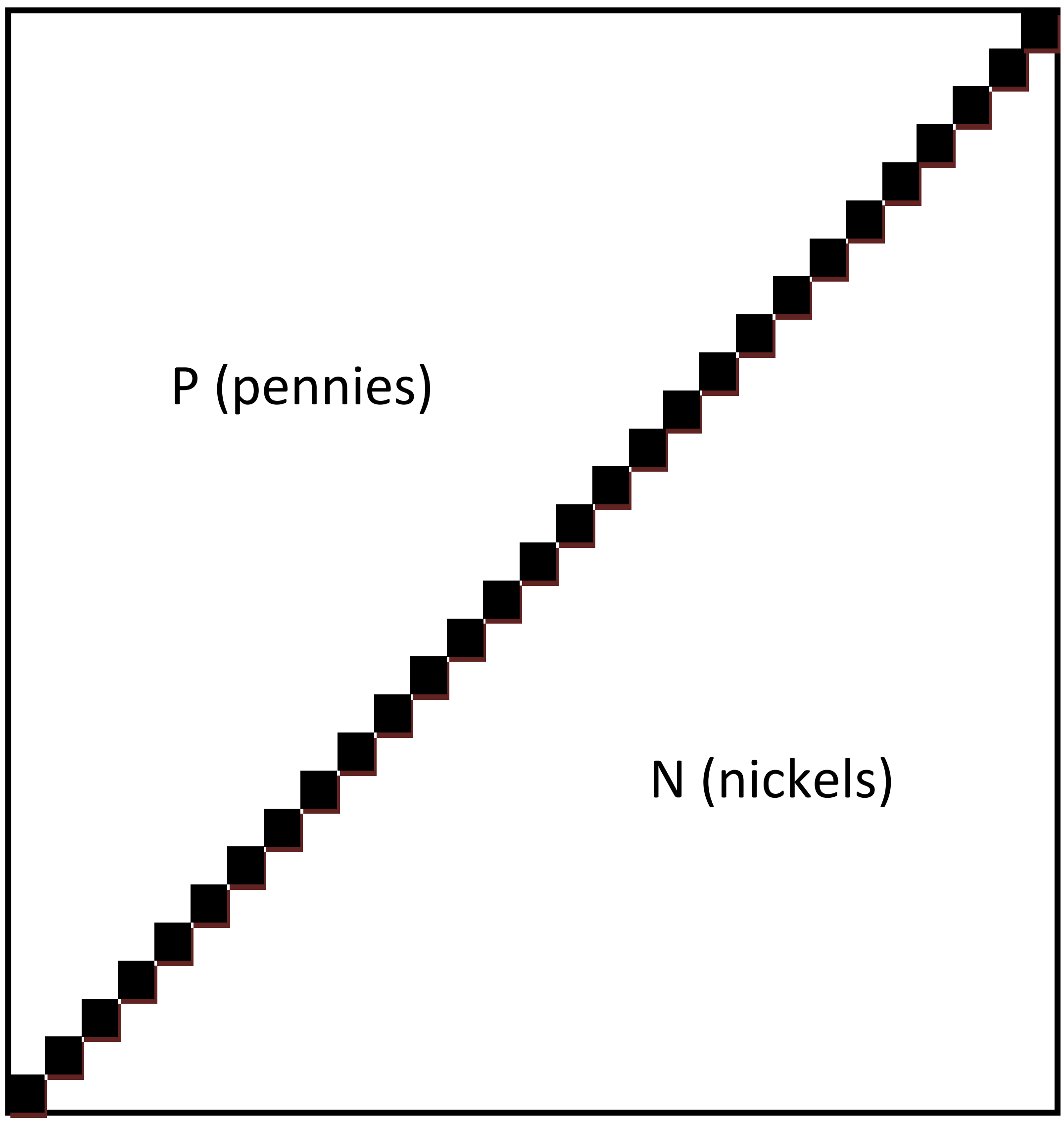
## Time to make connections:

P = pennies = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28

**Did you get the same answer for P before?**

**Question 7:** Did Gauss use a 100 x 100 chessboard to figure out 1 + 2 + 3 + 4 + … + 99 + 100 = ?

**Nothing prevents us from using a 100 x 100 chessboard!**




Q + P + N = T

- Q = 100
- P = N = ?
- T = $100^2$ =100 x 100 = 10,000

100 + P + P = 10,000

→ 100 + 2P = 10,000

→ 2P = 10,000 — 100 = 9,900

→ P = 9,900 : 2 = 4, 950.

But wait! Is this the answer which Gauss found out?

P = 1 + 2 + 3 + … + 98 + 99 = 4,950

Gauss added an extra 100:

1 + 2 + 3 + … + 98 + 99 + 100 = P + 100 = 4,950 = **5,050.**

## <u>Question 8</u>: How did Gauss really figure out the sum in his head?

**<u>Answer to Question 8</u>: Gauss noticed that adding up in a different order was much easier!**

For example:

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =$
$= (1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + 5 =$
$= 10 + 10 + 10 + 10 + 10 + 5 = 40 + 5 = 45$

Let’s do it on the whole big example:

$1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100 =$

$= (1 + 100) + (2 + 99) + (3 + 98) + (4 + 97) + \ldots + (50 + 51)$

$= 101 + 101 + 101 + 101 + \ldots + 101 = 50 \times 101 = 5,050.$

## <u>For the Die-Hards: A project to work on</u>

1. The formula for adding up all numbers from 1 to *n* is:

$$1 + 2 + 3 + 4 + \ldots + n = n\,(n+1)/2$$

2. Try to sum up all odd numbers from 1 to 99. What will you get? Is there a formula for all such sums?