Lesson 13, December,01, 2009

Overview

1. We started with a logic problem. Since the answers varied, we spent some time on the discussion of them and giving arguments for the right answers. As usual, after the discussion I draw the table with elimination of choices, and I still can not convince them that this is a very easy and straightforward way to organize the solution!
2. " Silent cooperation". I used to make this experiment in my calculus classes, just for recreation. It was curious to find out, what would be outcomes with our young mathematicians. As it is mentioned in the Mosteller's book, the most common answers are 1,3,7. My undergraduate students also liked to name 100 and 0. Look at the distributions that we got with *kids*:

At 6 pm group:

0,0,1,1,1,3,7,8,12,119,245,586, 1 000 000, 9,999999.9, IIIIIIIIIIIIIIIIIII

(the last one is 19 sticks)

At 7 pm group:

4,4,6,7,7,8,10,11,24,51,100,101,196,624,1 000 000 000

At 7 pm group I asked some kids to explain their choices. Here are the answers ☺ :

- 101, because I like 100 and I added 1.
- 4, because it is quite common number.
- 7, because I am 7 years old.
- 4, because my favorite number is 6, and I subtracted 2. (- And why did you subtract two? - Because it is an even number.)
- 8, because I like circles, and it has two circles.

3. "The Gnome story" is about uniform probability distributions: How to construct a probability space with 2, 3, 4 equally likely outcomes. The distribution for painting **houses** were

6 pm 2 red, 13 blue (!);

7pm 6 red 8 blue.

For tables

6pm 1 blue, 6 green, 2 red

7pm   5 blue 8 green 1 red

For chairs

6pm Red 2 Blue 7 Green 3 Yellow 3

7pm Red 1 Blue 5 Green 1 Yellow 7

Even for adult students it would be a significant challenge to connect a theory with practice. As you can see, our distributions are quite far from being uniform. So I did not make any connections at all and basically we just played. Let us leave the theory for the later in our lives!

4. At 6 pm group we went through the Question 1 smoothly. But at 7 pm there came out an opinion that numbers like 3 appear more often on a top of a die, because this is "a number in the middle", 1 also comes out quite often, but 6 almost never appears on the top. I made some pitiful attempt to explain that the cube is the same from each side, but I know that I did not sound convincing.

5. I also noticed that some kids first colored the pictures, and after that rolled the die (ok, who cares ☺). And two girls, who are very good friends, tried to color everything similarly, so they also have had to roll the die more than once. That may explain a little bit such a big fluctuation of our distribution from the uniform one, but that really does not matter - more was not expected from this lesson.

6. Finally, we held a discussion about random numbers in our lives .We used the handout with pictures. Here I made a huge mistake. On one of the pictures Santa Clause gives away presents out of the bag with the sign "Sweepstakes". It turned out, that I was awfully wrong: Santa Claus never gives the presents randomly, he gives them only to those who behave well during the year! Very inappropriate example!

PETS

Each of the four girls, Mina, Lisa, Kianna and Claire, has one of the four following animals: a cat, a dog, a gold fish and a canary-bird. Each of the girls has a different pet:

- Lisa has a pet with fur.
- Claire has a pet with four legs.
- Lisa and Mina don't like cats.
- Kianna's pet cannot swim.

What pet does each of the girls have?

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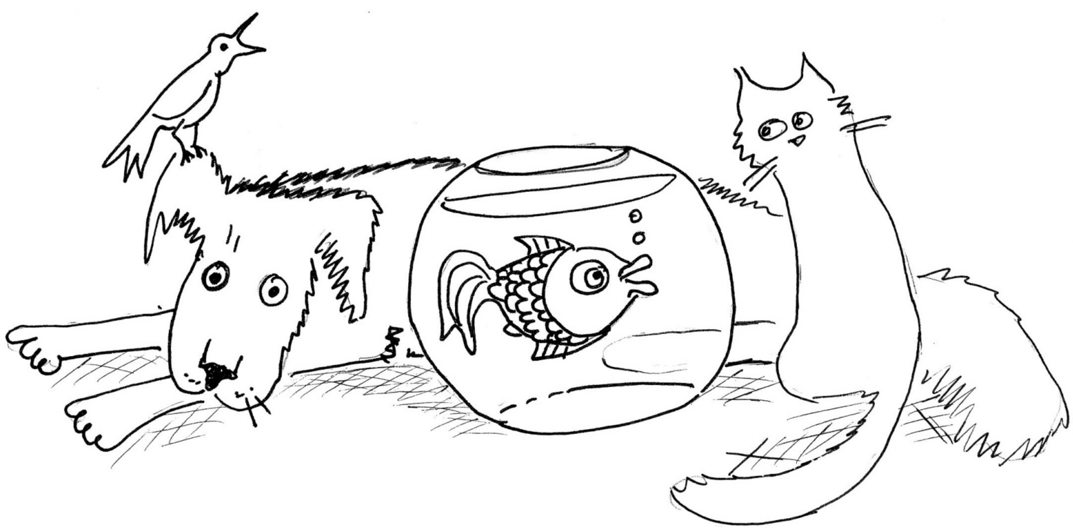
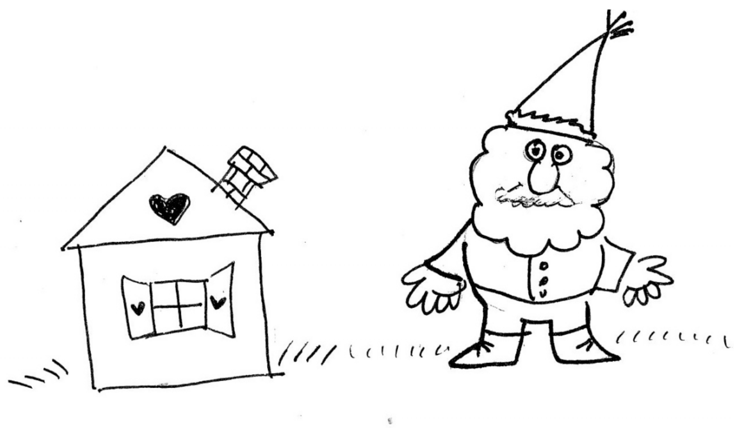
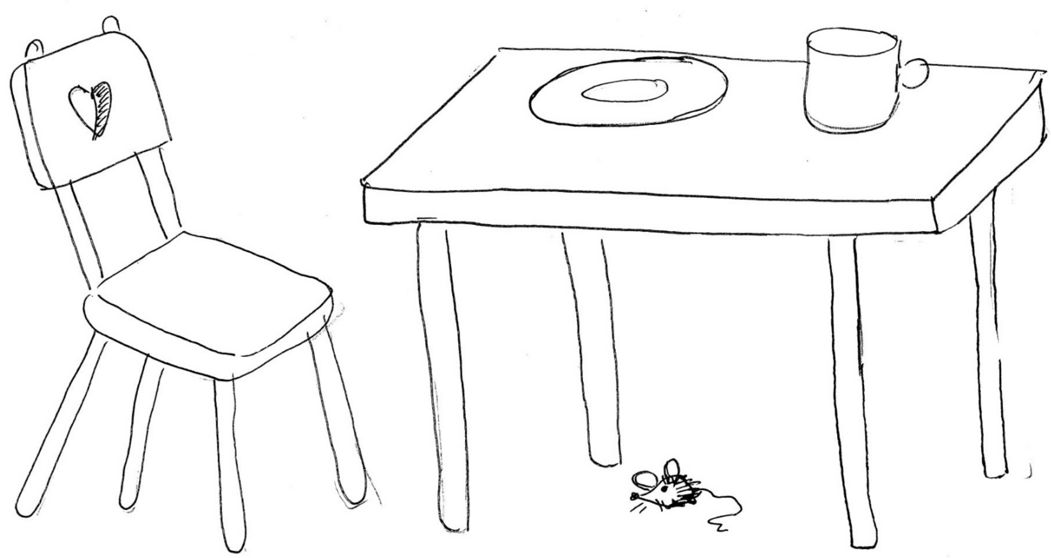






THE GNOMES PROBLEM

1. Once upon a time there lived a family of gnomes. They traveled around the world and finally came to a place where they decided to settle down and build a village. Every gnome

built a house for himself. They wanted to paint their houses, and they wanted to paint each house using only one color, but they only had paint in two colors: red and blue. The gnomes could not agree on how to choose the color for each of the house. So, they decided to do it by chance. Each gnome rolled a dice. When the number on the top was odd, the house was painted red. If the number on the top was even, the house was painted blue.

Question 1: Are the chances that each house was painted red or blue equal for both colors? Roll a dice and color the house.

2. There was a table in each of the houses. When the gnomes decided to paint the tables, they had paint in three different colors: blue, green and red. One of the gnomes said:

- Let us roll a dice again. If the number on the top is 1, we will use a blue paint. If the number is 2, we will use a red paint, in all other cases we will use a green paint.

Question 2: Are the chances that a table is painted in a color equal for each of the three colors? How to change the rule of choosing the paint so that the chances become equal? Roll a die and color the table on the picture according to your rule.

3. Every gnome had a chair that he wanted to paint. A chair could be painted in one of four colors: blue, green, yellow and red.

Question 3: Make a rule on how to determine what paint color to choose so that each of the four colors gets an equal chance of being picked. Color the chair on the picture according to your rule.

4. Every gnome decided to buy a plate and a mug. Mugs and plates were in three different colors: blue, red and green. One of the gnomes exclaimed: “Great! Every gnome in the village can have a different set of a mug and a plate!” (The colors of the plate and the mug each of the gnomes would buy did not have to match).

Question 4: If there were 20 gnomes in the village, was that gnome correct? (Hint: count the number of different pairs of mugs and plates there are).

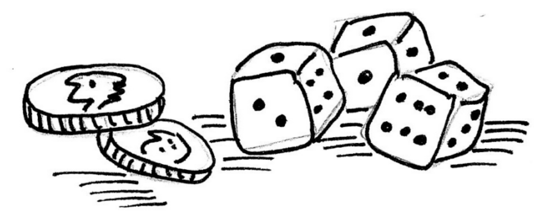
Question 5: Help the gnomes to make a rule on how to choose colors for their mugs and plates using a dice.

SILENT COOPERATION

This experiment was taken from F. Mosteller's Fifty Challenging Problems in Probability.

Two strangers are separately asked to choose one of the positive whole numbers and advised that if they both choose the same number, they both get a prize. If you were one of those people, what number would you choose?

RANDOM CHOICES WITH EQUAL CHANCES (UNIIFORM DISTRIBUTION)



A. What does the word "random" mean?

B. What are the situations where people may need random choices with equal (fair) chances for each choice to appear?

On the next page for each picture explain why people may want to make a random choice in the described situation.

Examples:

1. To choose a winner in a lottery.
2. To choose a product on a factory for a quality control .
3. To select out individuals for an unwanted task in a fair way
4. To select the order of games in a soccer tournament.
5. To choose a group of people to conduct a poll.

C. What are advantages and disadvantages of the following ways of random drawing:

1. Flip a coin.
2. Roll a die.
3. Draw a straw.
4. Ask a stranger on the street.
5. Ask a computer.

Remark for parents. The best source of random numbers is nature.

For modern ways to generate true random numbers check the information on the websites

http://www.random.org/randomness/,

http://www.fourmilab.ch/hotbits/