

# Equivalent or not?

0 1 2 3 4 5 6 7 8 9

Yesterday, when I was waiting for the bus, I heard some people talking. Here is what they said:

## **Person A**

“I say that 0 2 4 6 and 8 are equivalent to each other and that so are 1 3 5 7 and 9.”

## **Person B**

“No. I think 0 3 6 and 9 are equivalent to each other, 1 4 7 are equivalent, and so are 2 5 8.”

## **Person C**

“I used to think like you, but then I discovered that 0 4 6 and 9 are equivalent, so are 1 2 3 5 7, and 8 is not equivalent to anything but itself.”

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I was confused for a while, but when I got home, I realized they were all wrong:

0 4 5 and 9 are equivalent, 1 2 and 6 are, and so are 3 7 and 8.

## A. Equivalence Relations on the Set of People in this Room:

Questions:

Are you in the same equivalence class as your neighbors?

Do the equivalence classes have a simple characterization?

How many equivalence classes are there for this relation?

Did you need more data to answer the previous question?

1.  $S \sim T$  if the first name of  $S$  has the same number of letters as the first name of  $T$ .

2.  $M \sim N$  if  $M$  sits in the same row as  $N$ .

3.  $J \sim K$  if the year  $J$  was born has the same number of digits as the year  $K$  was born.

4.  $U \sim Z$  if  $U$  has the same number of names as  $Z$ .

5.  $X \sim Y$  if:

“the house number of  $X$ ”  $-$ (minus) “the house number of  $Y$ ”  $= 2k$ .

Here  $k$  is some integer. In other words, the difference of the house numbers is an even number.

6.  $D \sim E$  if .... find some interesting relations and let us know

**B. For the break** Is it raining outside now? How can you find out? How does this relate to equivalence relations (especially to Relation 5)?

**C. Given a partition, what is the equivalence relation?**

Using the objects in front of you, find out what persons  $A$ ,  $B$ ,  $C$  and me were thinking.

What are some other relations you can put on this set of objects?