More problems

1. Given positive numbers a, b, c, d, prove that
$$\frac{a^3+b^3+c^3}{a+b+c}+\frac{b^3+c^3+d^3}{b+c+d}+\frac{c^3+d^3+a^3}{c+d+a}+\frac{d^3+a^3+b^3}{d+a+b}\geq a^2+b^2+c^2+d^2$$

2. Prove that the given system of equations has exactly one solution.
$$\begin{aligned} x+y+z&=3\\ x^2+y^2+z^2&=3\\ x^3+y^3+z^3&=3\end{aligned}$$

3. Solve the system .
$$\begin{aligned} x^2+y^2&=2\\ x^3+y^3&=2\end{aligned}.$$

4. Express the given polynomial as a polynomial in the elementary symmetric polynomials.
   (i) x^n+y^n
   (ii) $x^n+y^n+z^n$
   (iii) $(x+y)(x+z)(y+z)$
   (iv) $xy^4+yz^4+zx^4+xz^4+yx^4+zy^4$.

5. Let a, b, c be real numbers such that $a+b+c=0$. Prove that
   (i) $$\left(\frac{a^2+b^2+c^2}{2}\right)\left(\frac{a^3+b^3+c^3}{3}\right)=\frac{a^5+b^5+c^5}{5};$$
   (ii) $$\left(\frac{a^2+b^2+c^2}{2}\right)\left(\frac{a^5+b^5+c^5}{5}\right)=\frac{a^7+b^7+c^7}{7}.$$
   (iii) Can you generalize the above equations?

6. Prove that the product of four consecutive terms of an arithmetic progression of integers plus the fourth power of the common difference is always a perfect square.