One-Player Games
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1. Begin with a stack of chips. On your turn, you may take either 2 or 3 chips from
the stack. If you take all the chips, you win; if there's 1 chip left, you lose. For
what starting numbers of chips is there a winning strategy?

What if you are only allowed to take 3 or 4 chips? What if you are only allowed
to take 5 or 11 chips? (In any case, if there's a pile of chips left at the end, you
lose, and if you take them all, you win).

2. [Thanks to A. Bogolmony of http://cut-the-knot.com for reminding me of this
idea] In the game of squares and circles, begin with a collection of squares and
circles. For example, you might start with three circles and a square. At each
step, cross out any two shapes. If the shapes you just crossed out are the same,
draw one square. If they are different, draw one circle. Eventually, there is only
one shape left. You win if the final shape is a circle. What can you say about
what happens in this game? What about with different starting situations?

3. In the previous problem, what if we have squares, circles, and triangles, and the
rule is that you may only cross out two different shapes and then draw the third
shape. Can you win (by ending with just one circle) if you start with three circles
and a square?

What about other starting situations, like four of each shape for example?

What about if you can reverse the rule when you wish, crossing out one shape and
drawing one each of the other two shapes?

What if the rules are changed so that when you cross out two different shapes you
then draw two copies of the third shape? The goal is to get all the shapes the
same. What starting situations enable you to eventually win?

4. The Mad Veterinarian [http://bumblebeagle.org/madvet/index.html, which
incidentally also has some great solution discussions] has three machines. One
converts a cat into two dogs and a mouse (or vice-versa): 1C <-> 2D 1M. A
second machine does 1D <-> 1C 1M, and a third machine does 1M <-> 1C 3D.
The general puzzle is to start with just one animal and replicate it: what’s the
fewest cats (more than one) that you can turn one cat into (with no mice or dogs
left around)? Or, even more generally, starting with one cat can you describe all
the combinations of animals you can end up with?
5. Here’s a two-player game for a change: Player 1 writes a sequence of ten positive integers. Then player 2 writes a + or – sign in each of the nine spaces between the integers. In the end, if the final numeric result is odd, player 1 wins, and if even, player 2 wins. Who should win this game, and how?

What if player 1 is given a bag with a certain collection of numbers, each of which can be used only once? For example, if they have a bag containing the numbers 1 through 12? 1 through 11? 1 through 10?

What if player 2 can use exactly one multiplication sign, and eight + or – signs, in the nine spaces?

What if player 2 gets exactly two multiplication signs (and 7 + or – signs)?

6. Another one-player game: Start with a stack of \( n \) boxes. At each move, as long as any stacks have more than one box, split one stack into two parts, say \( x \) boxes into \( y \) and \( z \), and score \( yz \) points. How should you split them in order to maximize your score? What is the maximum score for each \( n \)?

7. Coin-flipping: Begin with some number of coins, say four for example, and set them on the table in a line, with a given starting sequence like HHTH for example. At each move, you may flip any two adjacent coins. You win if the final arrangement of the coins is all heads.

8. Coin-splitting: Begin with an infinite strip of squares, and a penny on one spot. At each move, you may either split the penny (remove it and put a penny on each adjacent spot) or merge two pennies (remove two pennies with exactly one space between them and put one on the space between; in other words, undoing the splitting operation). You may have any number of pennies on a given spot (but each move only splits one penny or merges two pennies into one). Starting with one penny, can you split and merge to end up with just one penny on the board in a different spot? What different spots are possible?