1. The rectangle $MNPQ$ is inside the rectangle $ABCD$. The portion of the rectangle $ABCD$ outside of $MNPQ$ is colored in green. Using just a straightedge construct a line that divides the green figure in two parts of equal areas.

2. Given a triangle $ABC$ such that $\angle B = 90^\circ$, denote by $k$ a circle with center on $BC$ that is tangent to $AC$. Denote by $T$ a point of tangency of $k$ and the tangent from $A$ to $k$ (different from $AC$). If $B'$ is the midpoint of $AC$ and $M$ the intersection of $BB'$ and $AT$, prove that $MB = MT$.

3. If $a$, $b$, $c$ are positive real numbers prove that $a^2 + b^2 + c^2 \geq ab + bc + ca$.

4. If $a$, $b$, $c$ are positive real numbers prove that

$$
\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.
$$

5. Is it possible to tile $8 \times 8$ board with figures congruent to $\boxed{\text{}}$? What about $10 \times 10$ board?

6. Determine the maximal number of figures $\boxed{\text{}}$ that can be placed in a grid $6 \times 6$ without overlapping.

7. If a $5 \times n$ rectangle can be tiled with $n$ pieces congruent to $\boxed{\text{}}$ prove that $n$ is even.

8. Prove that for each $n \geq 3$, there are positive odd integers $x$ and $y$ such that

$$
2^n = x^2 + 7y^2.
$$

9. There are 12 students in a class and among them each two are either friends or enemies. Prove that it is possible to choose two and paint them in green such that the following condition is satisfied: Among the remaining 10 students, it is possible to choose 5 each of which is either mutual friend or mutual enemy to both of green students.

10. If $a$, $b$, $c$ are positive real numbers prove that

$$
\frac{a+b}{2b+c} + \frac{b+c}{2c+a} + \frac{c+a}{2a+b} \geq 2.
$$

11. The incircle of triangle $ABC$ touches sides $BC$, $CA$, and $AB$ at $A_1$, $B_1$, and $C_1$ respectively. Prove that the perpendiculars from the midpoints of $A_1B_1$, $B_1C_1$, and $C_1A_1$ to $AB$, $BC$, and $CA$ are concurrent.

12. A circle with center $O$ passes through points $A$ and $C$ and intersects the sides $AB$ and $BC$ of the triangle $ABC$ at points $K$ and $N$, respectively. The circumscribed circles of the triangles $ABC$ and $KBN$ intersect at two distinct points $B$ and $M$. Prove that $\angle OMB = 90^\circ$. 
