Bay Area Mathematical Olympiad
and Mathematical Circles
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Berkeley Math Circle
Monthly Contest 4
Due January 11, 2004

1. A candidate running for an office was distributing leaflets. At each campaign event he distributed half of all the leaflets left and another half-leaflet. If at the fifteenth event he ran out of leaflets, how many leaflets did he start with?

2. Let n be a positive integer. Consider an $n \times n$ table with entries $1, 2, \ldots n; (n+1), (n+2), \ldots 2n; \ldots; (n-1)n+1, \ldots n^2$ written in order starting top left and moving along each row in turn left to right. We choose n entries of the table such that exactly one entry is chosen in each row and each column. What are the possible values of the sum of the selected entries?

3. The if two heights of the triangle ABC, dropped from vertices A and C, intersect inside of the triangle and one of them is divided by the point of intersection into two equal parts, and the other one in the ration 2:1, counting from the vertex, what is the angle B of this triangle?

4. Forty students participated in a mathematical contest. They had 5 problems to solve. It is known that each problem was correctly solved by at least 23 participants. Prove that there must be two participants such that every problem was solved by at least one of these two students.

5. Find all functions $f : \mathbf{R} \mapsto \mathbf{R}$ that satisfy the inequality

$$\left| \sum_{k=1}^{n} 3^k (f(x-ky) - f(x+ky)) \right| < 1$$

for every positive integer n and for all $x, y \in R$.