Bay Area Mathematical Olympiad
and Mathematical Circles

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Berkeley Math Circle
Monthly Contest 1
Due September 28, 2003

1. a) One Sunday, Zvezda wrote 14 numbers in a circle, so that each number is equal to the sum of its two neighbors. Prove that the sum of all 14 numbers is 0.

b) On the next Sunday, Zvezda wrote 21 numbers in a circle, and this time each number was equal to **half** the sum of its two neighbors. What is the sum of all 21 numbers, if one of the numbers is 3?

2. A grasshopper lives on a coordinate line. It starts off at 1. It can jump either 1 unit or 5 units either to the right or to the left. However, the coordinate line has holes at all points with coordinates divisible by 4 (e.g. there are holes at -4, 0, 4, 8 etc.), so the grasshopper can not jump to any of those points. Can it reach point 3 after 2003 jumps?

3. The sets A and B and be form a *prtition* of positive integers if $A \cap B = \emptyset$ and $A \cup B = N$. The set S is called *prohibited* for the partition, if $k + l \neq s$ for any $k, l \in A$, $s \in S$ and any $k, l \in B$, $s \in S$.

a) Define *Fibonacci numbers* f_i by letting $f_1 = 1, f_2 = 2$ and $f_{i+1} = f_i + f_{i-1}$, so that $f_3 = 3$, $f_4 = 5$ etc. How many partitions for with the set F of all Fibonacci numbers is prohibited are there? (We count A, B and B, A as the same partition.)

b) How many partitions for which the set P of all powers of 2 is prohibited are there? What if we require in addition that $P \subseteq A$?

4. The circle ω is drawn through the vertices A and B of the triangle ABC. If ω intersects AC at point M and BC at point P. The segment MP contains the center of the circle inscribed in ABC. Given that $AB = c$, $BC = a$ and $CA = b$, find MP.

5. For which n is it possible to fill the n by n table with 0's, 1's and 2's so that the sums of numbers in rows and columns take all different values from 1 to $2n$?