Berkeley Math Circle Monthly Contest # 2

Due October 28, 2001

1. Oaz and Andrew have a bag with 101 candies in it. They decide to play a game. Andrew goes first, and on each turn they take a positive number of pieces of candy from the bag until all of the candy is gone. Neither gets all of the candy. Oaz wins if the number of candies that he has is relatively prime to the number of candies that Andrew has; otherwise, Andrew wins. Who has a winning strategy?

2. On a spherical planet called Zvezda the temperature varies continuously over the entire surface. The government needs to put two satellite control stations on diametrically opposite points on the planet. However, the control stations can only communicate if they are in areas of equal temperature. Prove that it is possible to build two control stations that can communicate.

3. Find all roots of
$$x^4 + x^3 + x^2 + mx + m^2 = 0$$
in rectangular form.

4. Prove for any cyclic quadrilateral $ABCD$
$$\frac{AC}{BD} = \frac{CD * CB + AB * AD}{DC * DA + BA * BC}\ .$$

5. The evil Inna has captured Neil and is making him do math problems. Every day she poses a challenge to Neil. If on day n he can arrange the numbers $1, 2, 3, \ldots, n^2$ into an $n \times n$ matrix so that the product of the numbers in each row and column is the same he can go free. Unfortunately, on the first day of his capture Neil was too stunned to work on the problem. Will he ever win free? If yes, on what day?