BAMO 2000: PRACTICE PROBLEMS

ZVEZDELINA STANKOVA-FRENKEL
MILLS COLLEGE

Note: You have 4 hours to solve as many problems as you can from the following list of 5 problems. Each solution should be written clearly and in detail on a separate sheet of paper. Each problem is worth 7 points. Partial credit will be awarded for partial solutions.

Problem 1. Alice plays the following game: she writes in one row all numbers 1,2,3,...,2001. She then starts placing the signs $+$ or $-$ between any two adjacent numbers. In the end, she calculates the value of the resulting expression and wins if this value is 0. Prove that Alice can *never* win.

Problem 2. Let $a_1,\ a_2, ..., a_n$ $(n \geq 2)$ be a sequence of integers. Prove that we can choose several of its members so that the sum of their squares is divisible by n.

Problem 3. On circle k we have 4 points P, A, Q and B. We draw the circles k_1 and k_2 so that they have diameters AQ and BQ, correspondingly. Let k_1 and k_2 intersect in point L (and Q). Line PA and circle k_1 intersect in point M (and A). Line PB and circle k_2 intersect in point N (and B). Prove that the three points N, L and M lie on a line (i.e. they are *collinear*.)

Figure 1. Problems 2 and 5

Problem 4. Prove that if α is a positive integer such that $3^\alpha - 2^\alpha$ is a power of a prime number, then α is also a prime number.

Problem 5. A lock has 25 keys arranged in a 5×5 array, each key oriented either horizontally or vertically. In order to open it, all the keys must be vertically oriented. When a key is switched to another position, all the other keys in the same row and column automatically switch their positions too (see diagram). Show that there exists an initial position of all keys, which cannot be unlocked. (Only one key at a time can be switched. Compare with Problem 3 from BAMO 1999.)