1 Number of Factors

1. How many factors (divisors) does the number 15 have (including 1 and itself)?

2. How many factors does the number 100 have?

3. More generally, if $p$, $q$, and $r$ are three different prime numbers, find the number of different factors of:

   (a) $pq$
   (b) $p^2q$
   (c) $p^2q^2$
   (d) $p^kq^m$
   (e) $p^kq^mr^n$

2 Greatest Common Factor and Least Common Multiple

- The Greatest Common Divisor (gcd) of two natural numbers is the greatest natural number that divides them both.

- The Least Common Multiple (lcm) of two natural numbers is the least natural number that is divisible by both of them.

4. Given numbers $x = 2^8 \cdot 5^3 \cdot 7$ and $y = 2^5 \cdot 3 \cdot 5^7$, find $gcd(x, y)$ and $lcm(x, y)$.

5. What is the $gcd$ and the $lcm$ of 2000 and 7200?

6. What is the $gcd$ and the $lcm$ of 847 and 539?

7. For how many values of $k$ is $12^{12}$ the least common multiple of the natural numbers $6^6$, $8^8$, and $k$?
3 Euclid’s Game


Start with two numbers in a box. Two players take turns writing a new number in the box that is the positive difference of two existing numbers in the box. The player that can no longer make a move wins.

8. Is there a winning strategy for Euclid’s Game? Does it depend on what the two starting numbers are?
9. What is \( \gcd(949, 2701) \)?
10. What is \( \gcd(451, 287) \)?
11. Reduce the fraction \( \frac{2023}{2431} \) to lowest terms.
12. Find the gcd of the numbers \( 2n + 13 \) and \( n + 7 \).
13. Prove that the fraction \( \frac{21n + 4}{14n + 3} \) cannot be reduced for any natural number \( n \). (1959 IMO Problem 1)
14. Find \( \gcd(111...111, 111....111) \), where the first number has one hundred 1’s and the second has sixty 1’s.
15. Find \( \gcd(2^{100} - 1, 2^{120} - 1) \)

Many of these problems are from Mathematics Circles: the Russian Experience by Fromkin, Genkin, and Itenberg.