

- (Bulgaria, 1997) Two ordered n -tuples (a_1, \dots, a_n) and (b_1, \dots, b_n) are *separated* if each n -tuple consists of n distinct elements of $\{1, \dots, n+1\}$, and there exist indices $i \neq j$ such that $a_i = b_j$. Find the maximum number of n -tuples such that any two of them are separated.
- (China, 1997) Let a_1, a_2, \dots be nonnegative real numbers such that

$$a_{n+m} \leq a_n + a_m \quad (m, n \in \mathbb{N}).$$

Prove that

$$a_n \leq ma_1 + \left(\frac{n}{m} - 1\right) a_m$$

for all $n \geq m$.

- (Czech/Slovak, 1997) Show that there exists an increasing sequence $\{a_n\}_{n=1}^{\infty}$ of natural numbers such that for any $k \geq 0$, the sequence $\{k + a_n\}$ contains only finitely many primes.
- (Iran, 1997) Suppose w_1, \dots, w_k are distinct real numbers with nonzero sum. Prove that there exist integers n_1, \dots, n_k such that $n_1 w_1 + \dots + n_k w_k > 0$, but for any permutation σ of $\{1, \dots, k\}$ besides the identity permutation, we have $n_1 w_{\sigma(1)} + \dots + n_k w_{\sigma(k)} < 0$.
- (Iran, 1997) Suppose $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a decreasing function such that for all $x, y \in \mathbb{R}^+$,

$$f(x+y) + f(f(x) + f(y)) = f(f(x + f(y)) + f(y + f(x))).$$

Prove that $f(f(x)) = x$.

- (Romania, 1997) Let P be the set of points in the plane and D the set of lines in the plane. Determine whether there exists a bijective function $f : P \rightarrow D$ such that for any three collinear points A, B, C , the lines $f(A), f(B), f(C)$ are either parallel or concurrent.
- (Russia, 1997) On an $m \times n$ checkerboard, where m and n are odd integers, are placed 1×2 dominoes so as to exactly cover all of the squares of the board except one corner square. It is permitted to slide a domino towards the empty square, thus exposing another square. Show that by a sequence of moves, any corner square of the board can be uncovered.
- (South Korea, 1997) In an acute triangle ABC with $AB \neq AC$, let V be the intersection of the angle bisector of A with BC , and let D be the foot of the perpendicular from A to BC . If E and F are the intersections of the circumcircle of AVD with CA and AB , respectively, show that the lines AD, BE, CF are concurrent.
- (Taiwan, 1997) Let $k = 2^{2^n} + 1$ for some positive integer n . Show that k is a prime if and only if k divides $3^{(k-1)/2} + 1$.

10. (Turkey, 1997) In a soccer league, when a player moves from a team X with x players to a team Y with Y players, the federation receives $y - x$ million dollars from Y if $y \geq x$, but pays $x - y$ million dollars to X if $x > y$. There is no limit to the number of times a player may move during a season. The league consists of 18 teams, each of which begins a certain season with 20 players. At the end of the season, 12 teams end up with 20 players, while the other 6 end up with 16, 16, 21, 22, 22, 23 players, respectively. What is the maximum amount the federation could have earned during the season?